ISSN 1745-8587

Birkbeck UNIVERSITY OF LONDON

School of Economics, Mathematics and Statistics

BWPEF 1008

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Sandeep Kapur
Birkbeck, University of London

March 2010

Birkbeck Working Papers in Economics & Finance

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Abstract

This note explores the link between anticipated information and a preference for liquidity in investment choices. Given a subjective ordering of investment portfolios by their liquidity, we identify a sufficient condition under which the prospect of finer resolution of uncertainty creates a preference for more liquid positions. We then show how this condition might arise naturally in some standard classes of sequential decision problems.

JEL Codes: D81, E41, G11

Keywords: Liquidity preference, information

*Department of Economics, Birkbeck, University of London, Malet St, London WC1E 7HX, United Kingdom. *email*: s.kapur@bbk.ac.uk

The conception of what contributes to "liquidity" is a partly vague one, changing from time to time and depending on social practices and institutions. The order of preference in the minds of owners of wealth ... however is quite definite and is all we require for our analysis of the behaviour of the economic system.

Keynes (1936)

1 Introduction

The recent financial crisis has, once again, drawn attention to the issue of liquidity. Illiquidity reflects frictions in markets, which tend to be glossed over in periods of ostensible prosperity, but less so in times of economic distress. In asset markets illiquidity can emerge for a variety of reasons. It might reflect the difficulty of locating buyers in environments that involve costly search for counter-parties. In thick markets it can arise due to informational asymmetries that make it hard for a seller to convince prospective buyers of the true value of assets offered for sale. At the onset of the current crisis, as interbank lending dried up, many financial institutions were said to be suffering from a liquidity crisis. In hindsight, in many cases the problem turned out to be that of insolvency rather than illiquidity.

The idea of liquidity and the notion of liquidity preference were major element in Keynes' analysis of the economic crisis of the 1930s. Indeed, Hicks (1982) thought that liquidity was a 'Keynesian word', in that it was Keynes who introduced the term into academic discourse. In his *Treatise on Money*, Keynes (1930) even attempted to define liquidity: he described one asset as more liquid than another 'if it is more certainly realisable at short notice without loss'. As the opening quote suggests, by the time it came to the *General Theory*, Keynes (1936) was less insistent on the need for precise characterization.

Hicks himself continued his quest for a suitable definition of liquidity for a long time: he thought that Keynes' original definition encompassed too many attributes of assets to generate an operational understanding of liquidity. Further, if liquidity depended on multiple attributes, any ordering of assets by their liquidity could only be a partial ordering. For Hicks the operative bit of the Keynesian definition was that the value of liquid assets is 'more certainly realisable'. Assets whose market price is time-varying, say, due to some exogenous uncertainty are less reliable stores of value than assets whose price is steady. The price trajectory may not be extraneous to the factors that prompt sale of the asset. Some assets are illiquid because the states of the world in which an owner would want to sell them are precisely the states in which others are reluctant to buy them. Residential property is illiquid in part because it is costly to locate buyers in a hurry but also because prices are likely to be depressed, due to correlated cyclical shocks in income, precisely at times when individuals may need to sell their house. The latter friction is particularly relevant for derivative assets such as mortgage-backed securities, where the problem is not so much one of locating potential buyers, but of realising the value of assets that have turned toxic.

Of course, there have been numerous attempts to construct objective measures of liquidity, at least in particular settings. Lippman and McCall (1986) considered a search-theoretic model in which the illiquidity of an asset relates to the costly wait for a buyer with sufficiently high willingness to pay. If buyers arrive by a random process there is a tradeoff between early sale at a possibly low price and the costly wait for a better offer. Lippman and McCall proposed an 'operational measure of liquidity' to capture this tradeoff in a single index: namely, the expected time taken to sell the asset under an optimal selling policy (one that maximizes the expected discounted value of the proceeds from sale). For them one asset is less liquid than another if it takes longer to sell under an optimal selling policy. While appealing at first glance, their measure can generate counterintuitive measures of liquidity. For instance, they end up concluding that an increase in search costs can *increase* the liquidity of assets, because higher search costs lower the expected time to sale by lowering the reservation price. By design, their construct ignores the fact that the quicker sale involves a lower realised price.

Our approach to liquidity follows that articulated by Hicks (1974). For Hicks, a preference for liquidity arises in contexts that involve sequential adjustments of choices in light of new information. Sequentiality matters when choices in one period constrain choices in later periods (say, if some initial choices are irreversible) or affect the cost of making subsequent choices (say, due to transaction costs incurred in adjusting portfolios). If choices across time were independent, an agent could react optimally to all currently-available information each period, with no particular reason to value liquidity. The sequential arrival of information matters because it provides the impetus to adjust choices. In this analysis investment in liquid assets like 'barren' money is valuable purely because it enables investors to respond more easily to new information. This approach had precursors in the earlier works of Marschak (1949) and Hirschleifer (1972), among others. Mukherji and Sanyal (1988) explored how uncertainty and irreversibility can generate positive demand for liquid assets with low or negligible rates of return.

Our enquiry goes beyond explaining why a preference for liquid assets might arise. Rather, we focus on how the intensity of this preference varies with the degree of uncertainty (or, more accurately, with the anticipated resolution of the uncertainty). It is often suggested that the great uncertainty that accompanies economic crises reinforces a preference for liquidity. In the spirit of this exercise, Jones and Ostroy (1984)explored the link between flexibility and what they called the variability of beliefs. Hahn (1990) explored this idea further in a model where the adjustment of financial portfolios is impeded by transactions costs. He proposed an index that measures the illiquidity of a portfolio in terms of what we would be willing to pay, *ex ante*, to escape future transactions costs. Clearly, this subjective measure of illiquidity will be larger for agents who believe that new information is more likely to prompt significant adjustments to their portfolios, and especially so for agents who face high transactions costs. This, he reasoned, would allow him to establish a link between preference for liquidity and the prospect of future information. However, as constructed, Hahn's illiquidity index turns out to a proxy of the value of anticipated information itself, rather than a means of discovering the relationship between information and liquidity. As Arrow (1995) noted in a closely related context, 'it is intuitively reasonable that greater the anticipated information the greater the flexibility desired', but 'it is rather hard, as it turns out, to define the concepts rigorously and get definite results'. Our paper offers a modest step in this direction.

2 A General Result

We explore the contention that the prospect of finer resolution of uncertainty creates a preference for more liquid portfolios. While this conjecture seems intuitively plausible, its specification requires some care. First, we need to pick the criterion for ranking different portfolios according to their liquidity. We could do this by specifying the transactions technology or describing how choices in one period constrain choices in later periods. We then need to show how liquidity interacts with the resolution of uncertainty. A fully-developed model along these lines would allow us to demonstrate the link between the information structure and a preference for liquidity.

We begin, however, with an approach that allows the ordering of choices by their liquidity to be a primitive relation. In doing so, we allow for the possibility, implicit in Keynes' remark, that such orderings may be purely subjective. Consider an agent who must choose a position x from set X of available choices. For $x', x'' \in X$, we define a binary relation $\succeq_\ell$, where $x' \succeq_\ell x''$ is to be understood as the agent considers x' to be at least as liquid as x''. We do not require that this ordering be complete; that is, we allow the possibility that an individual may not be able to rank all pairs of choices in X according to their liquidity. The pair $(X, \succeq_\ell)$ is a partially-ordered set if $\succeq_\ell$ is reflexive, transitive and anti-symmetric on X.[1] Further, given a weak order ('as liquid as') as the primitive binary relation, we can construct a strict ordering ('more liquid than') in the usual way: we denote the strict relation as $\succ_\ell$.

The temporal resolution of uncertainty is modeled as an *information structure*. The underlying uncertainty is described by a set of states of nature. Information arrives in the form of signals that, over time, rule out some of these states. An information structure is given by the set of states, the set of signals, and a 'theory' that relates the signals to the possibility of various states. Information structures differ in their precision or informativeness in a sense to be elaborated below. Let Y be the set of all available information structures, with typical element y. Assume that there exists an ordering $\succeq_i$ on set Y, where $y' \succeq_i y''$ is to be understood as y' is at least as informative as y''. Given a weak order ('as informative as') we can construct a strict ordering ('more informative than'), to be denoted as $\succ_i$. As for the liquidity ordering, the informativeness order is partial.

Consider a sequential decision problem in which the agent makes an initial choice $x \in X$ and then reacts optimally to information structure $y \in Y$. We assume that the individual's choice can be represented as maximizing the value of some function $V : X \times Y \to R$. For instance, $V(x, y)$ could be interpreted as the expected utility of a sequence of actions that

[1] Reflexivity and transitivity do not require elaboration; a binary relation $\succeq$ on a set Z is said to be anti-symmetric if $z' \succeq z''$ and $z'' \succeq z'$ imply $z' = z''$.

involve initial action x and then reacting optimally to signals generated by information structure y.

Any reasonable specification of the decision problem would place some restrictions on this value function. For instance, it is reasonable to posit that information is valuable in a weak sense, so that $V(x, y') \geq V(x, y'')$ for $y' \succeq_i y''$.[2] That greater informativeness is valuable does not by itself lead us to conclude that greater informativeness will induce the selection of more liquid positions. To see what is involved, consider the family of optimization problems parameterized by information structure y:

$$\max_{x \in X} V(x, y), \quad y \in Y. \tag{1}$$

Let $M(y)$ be the set of maximizers to problem (1) for any given y. That is,

$$M(y) \equiv \arg\max_{x \in X} V(x, y). \tag{2}$$

Our question then is, if y' is more informative that y'', can we rank choices in $M(y')$ and $M(y'')$ in terms of their liquidity? As we show below, what we require is that the relative advantage of greater liquidity be increasing in the precision of information, in effect, a complementarity between liquidity and informativeness.[3] We introduce the following restriction.

Definition 1 (Increasing differences) *Let $(X, \succeq_\ell)$ and $(Y, \succeq_i)$ be partially-ordered sets. A function $V : X \times Y \to R$ is said to have increasing differences in*

[2] We ignore here the possibility that finer information may be costlier to process or may impose psychic decision-making costs.

[3] In contexts where the objective function $V(x, y)$ is differentiable, and interior optima characterized by first-order conditions, comparative static exercises of this sort rely on the implicit function theorem. Specifically, if x and y were real-valued, and $V(x, y)$ was twice-differentiable, complementarity could be reduced to the restriction that the cross-partial V_{xy} be positive. Our treatment here does *not* require differentiability and so allows for greater generality.

its two arguments x and y if for all $x' \succeq_\ell x''$ and $y' \succeq_i y''$, we have

$$V(x',y') - V(x'',y') \geq V(x',y'') - V(x'',y'').$$

We have strictly increasing differences when each of the relations above is strict.

Proposition 1 follows directly from Topkis (1998).

Proposition 1 *Let $(X, \succeq_\ell)$ and $(Y, \succeq_i)$ be partially-ordered sets, and $V : X \times Y \rightarrow R$ be a real-valued function. Consider $x' \in M(y')$ and $x'' \in M(y'')$ for $y' \succ_i y''$. If $V(x,y)$ has strictly increasing differences in x and y, then it cannot be that $x'' \succ_\ell x'$.*

Proof: Suppose, contrary to the claim, we have $x'' \succ_\ell x'$. Consider the following string of inequalities:

$$0 \geq V(x'',y') - V(x',y') > V(x'',y'') - V(x',y'') \geq 0.$$

The first inequality follows from the assumed optimality of x' for information structure y' (namely that $x' \in M(y')$), the second *strict* inequality follows from strictly increasing differences (given $y' \succ_i y''$ and our supposition that $x'' \succ_\ell x'$) and the last inequality follows from the assumed optimality of x'' for y''. As one of the inequalities is strict, we have a contradiction. ■

In words, given strictly increasing differences in the value function, the prospect of less precise information cannot induce the choice of a more liquid position. The crucial restriction – increasing differences – is a sufficient condition to generate a weak ordering of optimal choices according to their liquidity. In the next section we show how such complementarity between liquidity and information might arise naturally in the context of sequential decision problems.

3 Liquidity and Information

We consider the simplest setting in which an agent must make choices in two periods, $t = 1, 2$. His choice $a \in A$ in period 1 is followed by $b \in B(a)$ in period 2. The assumption that the initial choice a affects the set of subsequent choices, $B(a)$, allows us to capture the idea that some initial positions are more liquid – or afford more flexibility – than others. In particular, we say that one initial choice is more liquid than another if it permits a larger set of subsequent choices. Formally,

$$a \succeq_\ell a' \quad \text{if and only if} \quad B(a') \subseteq B(a).$$

Note that any ordering based on the criterion of set inclusion will, in general, be incomplete so that not all pairs of initial choices can be ranked in terms of their liquidity. Special cases may permit complete orderings: for instance, if investment in all assets other than cash is irreversible, then the liquidity ordering of portfolios depends unambiguously on the cash held in them.

In our sequential decision problem the payoff to future choices is uncertain but the uncertainty is gradually resolved over time. We model uncertainty in the usual way, as possible realizations of a state of nature. For simplicity, we take the set of states, Ω, to be finite, with typical element ω_k. We focus on settings in which the payoff to any profile of choices (a, b) is separable across time:[4]

$$u(a, b, \omega) = u_1(a) + u_2(b, \omega). \tag{3}$$

After making the initial choice but prior to making the period 2 choice, some of the uncertainty is resolved. We model the arrival of information

[4]Note that we also rule out any uncertainty in period 1 payoff: that can be incorporated without affecting our results.

as follows: there exists a signal function $\sigma : \Omega \rightarrow R$ that assigns a real number $s \in S$ to every state in Ω. Formally, the information structure is given by a partition P of Ω. The typical element $p(s)$ of this partition comprises all states that generate the signal s. That is,

$$p(s) = \{\omega \in \Omega : \sigma(\omega) = s\}.$$

Put simply, a signal identifies some element of the partition in which the true state lies, ruling out other states. Beliefs over the set of states are updated in a Bayesian fashion. Let π_k denote the prior probability associated with state ω_k. The ex-ante likelihood of signal s is

$$q(s) = \left\{ \sum_k \pi_k : \sigma(\omega_k) = s \right\}.$$

The posterior probability of state ω_k conditional on signal s is :

$$\pi_{k|s} = \begin{cases} \frac{\pi_k}{q(s)} & \text{if } \sigma(\omega_k) = s \\ 0 & \text{otherwise.} \end{cases}$$

An information structure is fully specified by the set Ω, the associated priors $\{\pi_k\}_k$, the signal function σ, and the set S of signals. Where it causes no ambiguity, we will refer to an information structure by its associated signal function σ.

Given a signal s, in period 2 the agent chooses $b \in B(a)$ to maximize expected utility given updated beliefs. Define this optimized value, conditional on initial choice a and signal s, as

$$v_2(a, s) = \max_{b \in B(a)} \sum_k \pi_{k|s} u_2(b, \omega_k). \tag{4}$$

If so, the decision problem in the initial period is to choose a to maximize

$$V(a, \sigma) = u_1(a) + \sum_{s \in S} q(s) v_2(a, s). \tag{5}$$

The value $V(a, \sigma)$ is sensitive to the precision of anticipated information. As formalized by Blackwell (1951), one information structure σ' is said to be *more informative than* another σ if it induces a finer partition of the set of states. Formally, we say $\sigma' \succ_i \sigma$ if the partition P' induced by σ' refines partition P induced by σ.[5]

Our aim is to compare an agent's initial choices under alternative information structures. Consider $\sigma' \succ_i \sigma$. Let s_j denote a typical signal generated by the (coarse) information structure σ, and let s'_i be a typical signal generated by the (finer) information structure σ'. Consider the typical element of the coarse partition P:

$$p(s_j) = \{\omega \in \Omega : \sigma(\omega) = s_j\}.$$

If information partition P' refines P, there exists a set of signals s'_i generated by σ' such that

$$I(s_j) = \{s'_i : p(s_j) = \cup p'(s'_i)\}.$$

In effect, $I(s_j)$ is the set of signals under the more informative signal function σ' that are 'garbled' into a common signal s_j under σ. The Bayesian updating mechanism implies the following relationship:

$$\sum_{s'_i \in I(s_j)} q(s'_i) \pi_{k|s'_i} = q(s_j) \pi_{k|s_j}. \tag{6}$$

[5]Thus, signal function σ' is more informative than signal function σ if $\sigma(\omega) \neq \sigma(\omega')$ implies $\sigma'(\omega) \neq \sigma'(\omega')$ for any pair of states ω, ω', and $\sigma'(\omega) \neq \sigma'(\omega')$ for some pair of states ω, ω' with $\sigma(\omega) = \sigma(\omega')$.

How does informativeness affect the preference for liquidity? Recall that, by Proposition 1, greater informativeness results in a choice of weakly more liquid positions as long as the value function $V(a, \sigma)$ has strictly increasing differences in a and σ. We develop two examples to illustrate how such increasing differences can emerge in familiar settings.

3.1 Irreversible choices

Consider, first, an example in which an agent must choose from a discrete set of alternatives in each period. In period 1 the agent can invest his unit endowment in one of N productive assets (labeled $n = 1, 2, \ldots N$) or hold it as liquid cash (asset m). Assume that investment made in a productive asset in period 1 is strictly irreversible in the second period (the strict irreversibility can be relaxed somewhat without affecting our qualitative results). In contrast, cash holdings can be transformed costlessly into any productive asset in period 2. In terms of notation developed above, the set of initial choices is $A = \{1, 2, \ldots, N, m\}$. Given $a \in A$, the set of permissible second-period choices is

$$B(a) = \begin{cases} A & \text{if } a = m, \\ \{a\} & \text{otherwise.} \end{cases} \tag{7}$$

This structure captures, in an extreme fashion, the idea that cash holdings are more liquid than irreversible investment in productive assets. That is, $m \succ_\ell n$ for all n. The various productive investments n are all irreversible so cannot be ranked against each other in terms of their liquidity.

In keeping with our general setting, the payoff to any profile of choices (a, b) is uncertain and depends on the unknown state of nature ω:

$$u(a, b, \omega) = u_1(a) + u_2(b, \omega)$$

Prior to making his period-2 choice the agent receives a signal s that provides information about the relative profitability of various productive assets. The payoff to choosing initial action a when facing information structure σ is

$$V(a,\sigma) = u_1(a) + \sum_{s \in S} q(s) v_2(a,s), \tag{8}$$

where $v_2(a,s)$ is as defined in equation (4).

Proposition 2 *Given (7) and (8), the function $V(a,\sigma)$ has increasing differences in a and σ.*

A formal proof is in the Appendix but the intuition is straightforward. It is sufficient to compare initial choice $a = m$ with any one of the initial choices $a \in \{1, 2, \ldots N\}$. Consider $\sigma' \succ_i \sigma$. If the agent's choice is irreversible, information is of no value, so that $V(n,\sigma') = V(n,\sigma)$ for $n = 1, 2, \ldots, N$. On the other hand, holding liquid cash allows the agent to react to new information, and the agent cannot be worse off with a more informative set of signals: $V(m,\sigma') \geq V(m,\sigma)$.

Together these two imply $V(m,\sigma') - V(n,\sigma') \geq V(m,\sigma) - V(n,\sigma)$. Of course this only amounts to *weakly* increasing differences. Strictly increasing differences emerge whenever the finer information is more informative in a decision-relevant fashion: we require that there exist some $s'_i \in I(s_j)$ such that optimal second-period choice under s'_i differs from optimal choice under s_j.

3.2 Concavity and increasing differences

Increasing differences can be established for another broad class of sequential decision problems that involve a mild restriction on the utility specification $u(a,b,\omega)$. Assume the choice set in each period is a compact interval of the set of real numbers. Given $a \in A \subseteq R_+$, we specify

$$B(a) = \{b \in R_+ : b \leq a\}.$$

The value a may refer, for instance, to the amount of cash holdings in a portfolio, with the proviso that second period investment is constrained by available cash holdings.[6] Here the liquidity of initial portfolios is completely ordered by the amount of cash held.

As before, ex-ante the payoff to any profile of choices (a, b) is uncertain:

$$u(a, b, \omega) = u_1(a) + u_2(b, \omega).$$

The uncertainty is resolved partially prior to making the second-period choice so that the value of initial choice a when facing information structure σ is

$$V(a, \sigma) = u_1(a) + \sum_{s \in S} q(s) v_2(a, s), \tag{9}$$

where $v_2(a, s)$ is as defined in equation (4). For this case we have the following proposition.

Proposition 3 *If $u_2(b, \omega)$ is concave in b, the function $V(a, \sigma)$ has increasing differences in a and σ.*

Once again, the proof is the Appendix but some intuition can be provided here. In this setting the second-period optimization problem involves the constrained maximization of a univariate concave function. Conditional on receiving a signal, either the constraint does not bind or, if it does, the best constrained-response is independent of the received signal. This feature, it turns out, is sufficient for the result to carry through.

[6] This assumes that all second period choices need to be financed outright. If cash is required solely to finance expected 'margin requirements', cash holdings need only be a fraction of second-period investment. Our arguments can easily be modified to cover such cases.

Concavity is a commonly-used restriction in many economic models. Indeed, in many cases, the payoff function is linear (that is, weakly concave) in the choice variable, so concavity does not appear to be too stringent as a requirement. Once again, as stated, the proposition only establishes weakly increasing differences: mild additional context-specific restrictions can establish strictly increasing differences.

4 Conclusion

This paper examines how the precision of anticipated information affects the preference for liquidity. It is commonly asserted that the preference for liquid or flexible positions is stronger if new information is likely to bring about drastic revision of current beliefs. We identify a form of complementarity between liquidity and information orderings that is sufficient to establish such a relationship. As Proposition 1 shows, when this complementarity condition – the so-called increasing differences – holds, greater informativeness cannot induce the choice of less liquid positions.

We then demonstrate how such complementarity might arise in particular classes of sequential decision problems. Where the choice is between a discrete set of irreversible investments and liquid cash, greater informativeness increases the reward, other things equal, to holding liquid cash. Alternatively, when the choice set is a compact interval constrained by the amount of available liquid assets, mild restrictions on the payoff functions – in particular, concavity – can generate the crucial complementarity between liquidity and informativeness. While we develop these two classes of sequential decision problems as examples, other restrictions can generate the same complementarity in a variety of other contexts. Thus in identifying the crucial form of complementarity we provide a possible route to analysing the relationship between information and liquidity.

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A Appendix

Proof of Proposition 2: Consider $\sigma' \succ_i \sigma$. Note first that, given the strict irreversibility of choices $n = 1, 2, \ldots N$, information is of no value when $a = n$. It follows directly from (6) that $V(n, \sigma') = V(n, \sigma)$.

We claim, next, that $V(m, \sigma') \geq V(m, \sigma)$. This follows from the fact that one cannot be worse off with a more informative set of signals. Formally, following m in the initial period, let b_j^* be the optimal second period choice in response to some coarse signal $s_j \in S$ and let b_i^* be the optimal response to a finer signal $s_i' \in S'$. Let π_{kj} and π_{ki} denote the associated posterior beliefs over the set of states. By construction, if b_i^* is optimal given s_i' we have

$$\sum \pi_{ki} u_2(b_i^*, \omega_k) \geq \sum \pi_{ki} u_2(b_j^*, \omega_k). \tag{A.1}$$

Aggregating this over all $s_i' \in I(s_j)$ and using (6), we have

$$\sum_{s_i' \in I(s_j)} q(s_i') \sum \pi_{ki} u_2(b_i^*, \omega_k) \geq q(s_j) \sum \pi_{kj} u_2(b_j^*, \omega_k). \tag{A.2}$$

Aggregating this over all $s_j \in S$, we have $V(m, \sigma') \geq V(m, \sigma)$.

Lastly, if the greater informativeness is decision-relevant – that is, for some s_j and $s_i' \in I(s_j)$ we have $b_i^* \neq b_j^*$ – then inequality (A.1) above is strict, and consequently we have $V(m, \sigma') > V(m, \sigma)$. ■

Proof of Proposition 3. Consider $\sigma' \succeq_i \sigma$. Here the liquidity ordering is given by the natural order on set A. We need to show that if $u_2(b, \omega)$ is concave in b, then for $a' \geq a$, we have

$$V(a', \sigma') - V(a, \sigma') \geq V(a', \sigma) - V(a, \sigma). \tag{A.3}$$

We have

$$V(a, \sigma) = u_1(a) + \sum_{s \in S} q(s) v_2(a, s), \tag{A.4}$$

where

$$v_2(a, s) = \max_{b \le a} E_{\omega|s} u_2(b, \omega). \tag{A.5}$$

For a given pair of initial choices a' and a, with $a' > a$, define

$$h(s) = v_2(a', s) - v_2(a, s) \tag{A.6}$$

to be the benefit, conditional on receiving signal s, of choosing the more liquid position a' relative to a. A larger constraint set can only improve the outcome, so $h(s) \geq 0$.

If σ' is more informative than σ, then every signal $s_j \in S$ is associated with a set $I(s_j) \subset S'$ of refined signals. Given (A.4), (A.5) and (A.6), a sufficient condition for (A.3) is that

$$\sum_{s_i' \in I(s_j)} q(s_i')h(s_i') - q(s_j)h(s_j) \geq 0 \quad \text{for all } s_j \in S. \tag{A.7}$$

We check that this condition holds for various cases. Define b_j^* be the optimal unconstrained response to signal a typical coarse signal s_j and similarly b_i^* for finer signal s_i'. We consider three cases.

Case I. Let signal s_j be such that $b_j^* \leq a < a'$, so that neither constraint binds: if so, $h(s_j) = 0$. Given that $h(s_i') \geq 0$, condition (A.7) holds for this case.

Case II. Suppose, next, that $a < b_j^* \leq a'$ so that constraint a binds under s_j but a' does not. Given concavity of the objective function, if a binds, the best constrained response is precisely $b = a$. If so,

$$h(s_j) = \sum \pi_{k|s_j} [u_2(b_j^*, \omega_k) - u_2(a, \omega_k)].$$

To save on notation, we write $h(s_j) = E_j[\tilde{u}_2(b_j^*) - \tilde{u}_2(a)]$, where the expectation is with respect to beliefs conditioned by signal s_j. For Case II, we consider three sub-cases, depending on possibilities for b_i^* in response to typical signal $s_i' \in I(s_j)$.

$$h(s_i) = \begin{cases} 0 & \text{if } b_i^* \leq a < a' \\ E_i[\tilde{u}_2(b_i^*) - \tilde{u}_2(a)] & \text{if } a < b_i^* \leq a' \\ E_i[\tilde{u}_2(a') - \tilde{u}_2(a)] & \text{if } a < a' < b_i^*. \end{cases}$$

We partition the set of signals $I(s_j)$ into $I_1(s_j)$, $I_2(s_j)$ and $I_3(s_j)$ to correspond to the three sub-cases. Then, using (6) expression (A.7) reduces to

$$\sum_{I_1} q_i E_i[\tilde{u}_2(a) - \tilde{u}_2(b_j^*)] + \sum_{I_2} q_i E_i[\tilde{u}_2(b_i^*) - \tilde{u}_2(b_j^*)] + \sum_{I_3} q_i E_i[\tilde{u}_2(a') - \tilde{u}_2(b_j^*)] \tag{A.8}$$

For sub-case 2 (that is, if $s_i' \in I_2(s_j)$), b_i^* is the optimal response to signal s_i', so must provide an expected payoff at least as large as any other response b_j^* (and strictly larger is $b_i^* \neq b_j^*$). If so, the second summation in the above expression is necessarily non-negative. For sub-case 1 (that is, if $s_i' \in I_1(s_j)$), we have $b_i^* \leq a < a'$, so $E_i\tilde{u}_2(x)$ is monotonically decreasing in the interval (a, a'). As b_j^* is assumed to lie in that interval, $E_i[\tilde{u}_2(a) - \tilde{u}_2(b_j^*)]$ is non-negative. A similar argument proves the non-negativity of the expression in sub-case 3.

Case III: $a < a' < b_j^*$. This case is handled like Case II: consider the three sub-cases for $s_i' \in I(s_j)$ and check that the left hand side of (A.7) is non-negative.

While this argument establishes only weakly increasing differences, if σ' is strictly more informative than σ, and if this greater informativeness is decision-relevant – that is, there exists some pair s_j and $s_i' \in I(s_j)$ where $b_i^* \neq b_j^*$ – then inequality (A.8) is strict, establishing strictly increasing differences.