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**Wage subsidies and international  
trade: When does policy coordination  
pay?**

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# Wage subsidies and international trade: When does policy coordination pay?\*

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## Abstract

National labour market institutions interact across national boundaries when product markets are global. Labour market policies can thus entail spill-overs, a fact widely ignored in the academic literature. This paper studies the effects of wage subsidies in an international duopoly model with unionised labour markets. We document both positive and negative spill-over effects and discuss the benefits and costs from international policy coordination both for the case of symmetric and asymmetric labour market institutions. Our results suggest that institutional differences could sign responsible for the slow speed at which labour market policy coordination has progressed so far.

*Keywords:* Wage subsidies, policy spill-overs, international policy coordination, unionised labour markets, trade, asymmetric labour market institutions

*JEL Classification:* F16, F42, J38, H87

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# 1 Introduction

National labour markets interact across national boundaries when product markets are global. Labour market policies can then have spill-over effects, which suggests that there are benefits from international policy coordination. Surprisingly, the academic literature has widely neglected the international effects of national labour market policies and the benefits (and potential costs) of coordinating them. Our paper is a first attempt to fill this gap. We study the effects of wage subsidies in an international duopoly model with unionised labour markets and discuss the costs and benefits from policy coordination both for the case of symmetric and asymmetric labour market institutions.

In the international policy arena, attempts have been made to coordinate policy making in many areas such as monetary policy, competition policy or product standards. Labour market policies, in contrast, remain mostly in national hands. The European Union (EU) is both a prime candidate for coordinative action and an example for the slow progress labour market policy coordination has made so far. In fact, the Amsterdam Treaty of 1997 asks all member states to treat employment 'as a matter of common concern and [to] coordinate their action' (European Union, 1997). Against this background the EU has designed an open method of coordination under which member states formulate common policy targets on the EU level but national governments design and implement appropriate policies to reach these targets.<sup>1</sup> Therefore, the interpretation of coordination in the area of labour market policies remains much weaker than in other policy areas.

Our analysis focuses on wage subsidies, which are an increasingly popular policy tool in many industrialised countries.<sup>2</sup> We construct a simple two-country trade model where product markets are linked through costly intra-industry trade. Each country hosts one firm that produces a horizontally differentiated good. In our model setup, imperfections in both the labour and the product market provide a rationale for subsidising wages. Labour markets are unionised and thus the prevailing wage rate is above its market-clearing level. In addition, governments pay unemployment benefits that increase the outside option of unions in the wage bargain. Finally,

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<sup>1</sup>One exception is the 'European Globalisation Adjustment Fund' that finances active labour market policies for workers who have been made redundant as a result of the globalisation process.

<sup>2</sup>Wage subsidies have been implemented in a number of countries, e.g. in Germany (Kombi-Lohn), Great Britain (Working Families Tax Credit) and the United States (Earned Income Tax Credit, EITC). It has been argued that, whereas traditional unemployment benefit systems increase the moral hazard problem and may prolong unemployment spells (see, e.g., Krueger and Mueller, 2010, for recent evidence), active labour market policies may be able to increase job-search incentives of the unemployed (see, e.g., Layard et al., 2005).

high entry barriers lead to imperfect competition in the product market and thus to inefficiently low levels of production and employment. Wage subsidies respond to these market imperfections by driving a wedge between the wage received by workers and the labour costs paid by firms. Such a policy reduces production costs and fosters labour demand without creating working poverty (Orszag and Snower, 2003).

Even though rigid labour market institutions lead to inefficiently high unemployment they are often politically infeasible to relax quickly (Saint-Paul, 2000). Since these institutions have emerged over years of bargaining between interest groups and governments, they can usually be changed only gradually and with considerable delay. To make our analysis especially relevant to policy makers, we therefore take the imperfections in the labour (and product) markets as given and examine the best policy response with these imperfections in place. In our model policymaker set wage subsidies so as to maximise welfare (subject to a budget constraint). Governments can either set wage subsidies unilaterally or coordinate their policies to maximise joint welfare. For most parts of the paper we consider a specific (and particularly popular) form of policy coordination where governments harmonise their policies and choose a common subsidy level. We then relax this assumption and allow for cooperating governments to agree on country specific subsidy policies.

Even in our simple model wage subsidies paid to domestic producers have a wide range of effects - in both the domestic and the foreign country. First, wage subsidies increase domestic wages and employment but decrease them abroad. Second, wage subsidies increase profits of the domestic firm but decrease profits abroad. Third, wage subsidies reduce consumer prices and thus increase consumer surplus at home and abroad. The relative strength of these effects is determined by the degree of product differentiation. If, for instance, the two firms produce close substitutes, wage subsidies will have a stronger (negative) spill-over effect on foreign employment, wages and profits.

National governments ignore the spill-over effects of wage subsidies when they decide unilaterally about the optimal subsidy levels. With symmetric labour markets international policy coordination then always increases welfare in both countries. With asymmetric labour markets, however, cross-border policy harmonisation entails costs. In particular, cooperating governments restrict their capability to account for the national features of their countries' institutions. Even more, with asymmetric labour market institutions costs and benefits of policy harmonisation are unevenly distributed among trading partners. In fact, a common subsidy level can benefit one country at the expense of the other. Policy harmonisation can hence

be difficult to enforce politically - even if it increases aggregate welfare. Our analysis thus provides a rationale for why labour market policies are still mostly in national hands although cooperation might be beneficial from a bird's eye perspective.

We consider two extensions of our benchmark model. First, we allow cooperating governments to choose country-specific subsidy levels and analyse the optimal cooperative policy. Since policy-makers then have two instruments at hand, aggregate welfare must be at least as high as if a common subsidy level were chosen. Nevertheless, a country on its own might still prefer the (globally sub-optimal) harmonisation policy. So even if policy cooperation could indeed be designed optimally in the real world, countries can still disagree about which policy to implement. As a second extension, we analyse governments that target employment rather than welfare. After all, the prime motive for subsidising wages is usually to stimulate employment (while adhering to some budget constraint). We demonstrate that under an employment target governments do not account for the positive effect of wage subsidies on consumer surplus, profit and labour income and thus set subsidy levels that are too low from a welfare perspective.

The academic literature has discussed strategic interactions between governments and the resulting benefits from coordination with respect to a wide range of policy issues - such as capital taxation, environmental regulations, and labour standards - but has widely neglected labour market policies. As an exception, Franzese and Hays (2006) provide empirical evidence for the European Union that an increase in expenditure on labour market training programmes in one country decreases spending by its neighbours. They argue (without a formal model) that their results could be explained by agglomeration effects along European border regions. For instance, effective training policies in the French border region to Belgium may attract Belgium workers and could enhance the pool of workers available for both Belgium and French employers. Labour market training then entails positive externalities and provides incentives for (neighbouring) countries to free-ride on them. In contrast, policy spill-overs in our model arise through product market linkages and thus even in the absence of cross-border labour movements.<sup>3</sup> Moreover, we demonstrate that the existence of spill-over effects does not necessarily justify (any form of) policy coordination, let alone make coordination politically enforceable.

More generally, our work is related to the strategic trade literature as pioneered by Brander and Spencer (1984) and Brander and Spencer (1985).<sup>4</sup> It is worth noting

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<sup>3</sup>Beyond doubt, in case of the European Union the free movement of labour can induce further spill-over effects not captured in our model. However, even within the EU labour mobility is still comparably low (see Zaiceva and Zimmermann, 2008).

<sup>4</sup>The basic structure of the underlying 'reciprocal dumping model' is described in Brander (1981).

that in these models wage subsidies will have effects similar to a general production subsidy to domestic firms if labour is the only input in production. More recently, models of international oligopoly have also been used to analyse the choice of the international commodity tax base (see, e.g., Haufler et al., 2005 and Haufler and Pflüger, 2007). A negative commodity tax collected under the origin principle (where taxes are collected in the country of production) will effectively be identical to a wage subsidy if labour is the only production factor. In fact, both wage subsidies and commodity taxes drive a wedge between the wage rate and the labour costs faced by firms.

Despite these strong analogies between wage subsidies on the one hand and production subsidies and commodity taxes on the other hand, our analysis differs in four important aspects from the classical strategic trade literature. First, we analyse wage subsidies in an international duopoly model with unionised labour markets along the lines of Naylor (1998).<sup>5</sup> Since labour market imperfections are the prime motive for active labour market policies, the question at hand can only be tackled by modelling them explicitly. Second, in our model both wages and employment are endogenous and labour income enters the welfare function that is maximised by governments. The strategic trade literature, in contrast, typically abstracts from labour income effects and considers changes in profit income and government revenues only. Third, we consider cross-country differences in national labour market institutions, namely differences in the generosity of national unemployment benefits. Such differences turn out to be crucial for the costs and benefits from policy coordination. Fourth, we do not only consider welfare but also the value of employment as a government target. After all, wage subsidies are most often promoted as a means to stimulate employment (while preventing take-home wages to fall) rather than to increase welfare overall.

The remainder of the paper is organised as follows. The basic model setting is presented in the next section. Sections 3 and 4 analyse the optimal responses of firms and trade unions, respectively, to given subsidy policies. Section 5 derives and compares the optimal welfare maximising subsidy levels under both policy competition and coordination. The section also distinguishes between symmetric and asymmetric labour market institutions. Section 6 discusses extensions of our benchmark model. Finally, Section 7 offers some concluding remarks.

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<sup>5</sup>Naylor (1998) does not study government interventions. Recently, Moriconi and Sato (2009) have analysed the effects of commodity taxation in a model with downward wage rigidity. However, their work assumes perfect competition in the product market and abstracts from cross-country differences in the labour market.

## 2 The Model

There are two countries, Home (H) and Foreign (F), where variables referring to the latter are marked by an asterisk (\*). Each country is endowed with a continuum of immobile consumers of unit measure 1.<sup>6</sup> Utility of consumers is a quasilinear function of a numeraire good  $m$  and the two differentiated products  $X$  and  $Y$ :

$$U(x, y, m) = a(x + y) - \frac{1}{2}(x^2 + y^2 + 2exy) + m, \quad (1)$$

$$U^*(x^*, y^*, m^*) = a(x^* + y^*) - \frac{1}{2}(x^{*2} + y^{*2} + 2ex^*y^*) + m^*, \quad (2)$$

where  $e \in [0, 1]$  is an inverse measure of product differentiation, and  $x$  and  $y$  denote consumption of good  $X$  and  $Y$ , respectively. Production of the differentiated goods  $X$  and  $Y$  is country-specific. Good  $X$  is produced by firm 1, which is located in Home, while good  $Y$  is produced by the foreign-based firm 2.

Each consumer supplies inelastically one unit of labour to the differentiated good sector,<sup>7</sup> is endowed with  $Z$  units of the numeraire good (which describes the individual endowment of non-labour assets), and may also receive an identical share of profit income. Since a country-specific trade union raises wages above the market clearing wage in each country, labour markets do not clear and workers can either be employed or unemployed. Employed workers (marked with the subscript  $EM$ ) earn a wage  $w$ , while unemployed workers ( $UN$ ) receive unemployment benefits of size  $b$ . Denoting profits by  $\Pi$  and also allowing for the possibility of lump-sum taxes  $h$ , the per-capita income of a type  $i$  worker ( $i = EM, UN$ ) can be written as:

$$I_{EM} = w + \Pi + Z - h, \quad I_{UN} = b + \Pi + Z - h, \quad (3)$$

$$I_{EM}^* = w^* + \Pi^* + Z^* - h^*, \quad I_{UN}^* = b^* + \Pi^* + Z^* - h^*. \quad (4)$$

Consumers spend their income on consumption of the differentiated varieties  $x$  and  $y$  and the numeraire good  $m$ . The budget constraint of a type  $i$  worker in Home

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<sup>6</sup>Assuming labour to be immobile is a serious model limitation only if there are sizeable labour flows between the two countries. In the past, labour mobility within Europe has been rather low compared to for example the US (Zaiceva and Zimmermann, 2008) and we shall see our model to be especially applicable to the former. Nevertheless, labour mobility could become more important in the near future. Allowing for migration is therefore an important extension to our analysis but one we leave for future research.

<sup>7</sup>As in e.g. Moriconi and Sato (2009) we assume that the numeraire is not produced using labor. Alternatively, one may think of workers being tied to the differentiated good sector due to e.g. sector-specific human capital.



and Foreign can then be written as:

$$I_i = p_x x + p_y y + m_i, \quad I_i^* = p_x^* x^* + p_y^* y^* + m_i^*, \quad (5)$$

where  $p_x$  and  $p_y$  are the prices of good  $X$  and  $Y$ .

In what follows, we will assume that consumers' endowments of the numeraire good are always large enough to guarantee positive demand for the numeraire good. Profit maximisation then yields the following linear demand functions:<sup>8</sup>

$$p_x = a - (x + ey), \quad p_y = a - (ex + y), \quad (6)$$

$$p_x^* = a - (x^* + ey^*), \quad p_y^* = a - (ex^* + y^*). \quad (7)$$

On the supply side, the two firms are assumed to compete as Cournot duopolists in segmented markets. Firms incur symmetric trading costs of  $t$  per unit of exports. Trading costs are exogenously given and should reflect a wide range of costs, including, for instance, transportation expenditures or costs of border formalities. Since we focus on trade between industrialised countries - and in particular on intra-EU trade - we assume that the two countries do not impose any revenue-generating tariffs.

In order to produce one unit of the good each firm has to employ one unit of labour. The governments of Home and Foreign pay a wage subsidy to their domestic firm of size  $s$  and  $s^*$ , respectively, per unit of labour employed. Unit production costs of firm 1 and 2 are thus given by  $w - s$  and  $w^* - s^*$  and profits of the two firms read:

$$\Pi = (p_x - w + s)x + (p_x^* - w + s - t)x^*, \quad (8)$$

$$\Pi^* = (p_y^* - w^* + s^* - t)y + (p_y - w^* + s^*)y. \quad (9)$$

Firms choose their level of production for the two markets so as to maximise profits. In doing so, they take the output choices of their competitors as given.

Following Naylor (1998) wage rates in both countries are set by a country-specific monopoly union that represents all the workers employed by the respective firm.

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<sup>8</sup>With quasi-linear preferences all income effects are swept up by the numeraire good and demand for the differentiated products is independent of aggregate income. A relevant path for further research is thus the integration of the model into a framework of General Oligopolistic Equilibrium as proposed by Neary (2002, 2003). In such a framework, wage subsidies could also affect aggregate employment through changes in net labour income (and not only by reducing labour costs).

Union objectives are given by the following Stone-Geary-type utility functions:

$$\Omega = (w - b)L, \quad \Omega^* = (w^* - b^*)L^*, \quad (10)$$

where  $L = x + x^*$  and  $L^* = y + y^*$  are the employment levels of firm 1 and 2, respectively. Unions maximise insider rents taking the unemployment benefit received by an unemployed worker as the outside option in the wage bargain. We assume that unions do not collude but set the optimal wage rate taking the other union's wage demand as given. Firms retain their right-to-manage and can choose their respective employment levels.

Governments in the two countries finance wage subsidies  $sL$  and unemployment benefits  $(1 - L)b$  by levying a lump-sum tax  $h$  on its population. The respective budget constraints therefore read:

$$h = sL + b(1 - L), \quad h^* = s^*L^* + b^*(1 - L^*). \quad (11)$$

Policy makers take the imperfections in the labour (and product) markets as given and implement the best policy response with these imperfections in place. In particular, due to political constraints policy makers take the country-specific levels of unemployment benefits ( $b$  and  $b^*$ ) as given and set wage subsidies in order to maximise welfare subject to the budget constraint.<sup>9</sup> An alternative target function, based on (the value of) employment, is discussed in Section 6.2. National welfare  $W$  is defined as the weighted average of (indirect) utilities of employed and unemployed workers, respectively:

$$\begin{aligned} W &= LU(x, y, m_{EM}) + (1 - L)U(x, y, m_{UN}) \\ &= \frac{x^2 + y^2}{2} + exy + \Pi + Lw + (1 - L)b - h + Z, \\ W &= L^*U^*(x^*, y^*, m_{EM}^*) + (1 - L^*)U^*(x^*, y^*, m_{UN}^*) \\ &= \frac{(x^*)^2 + (y^*)^2}{2} + ex^*y^* + \Pi^* + L^*w^* + (1 - L^*)b^* - h^* + Z^*. \end{aligned} \quad (12)$$

$$(13)$$

Notice that welfare can also be interpreted as the expected utility of a worker before her employment status is determined. Welfare increases with consumer surplus (as given by the first two terms in equations 12 and 13) and with the net income re-

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<sup>9</sup>As it turns out, welfare in the model depends on the difference between subsidy level and unemployment benefit. Governments thus implicitly choose the optimal difference  $s - b$  taking  $b$  as given. If, instead, governments could choose both  $s$  and  $b$ , the equilibrium values of  $s$  and  $b$  would be indeterminate because the optimal  $s - b$  could be reached by an infinite number of combinations of  $s$  and  $b$ . We interpret the level of unemployment benefits as an expression of preferences that change only slowly over time.

ceived by an average worker. Governments can either set wage subsidies unilaterally (taking the policy of the trading partner as given) or coordinate their policies to maximise joint welfare. We first assume that cooperating governments harmonise wage subsidies to a common level. In Section 6.1, however, we relax this assumption and discuss an optimal cooperation policy.

We consider a game of perfect information with the following timing of events:

1. The two governments set (non-cooperatively or cooperatively) the level of wage subsidies in the two countries.
2. Each union chooses a wage rate.
3. Each firm chooses its output (and thus employment) levels for the two product markets (home and abroad).

The game is solved by backward induction starting with the last stage of the decision making process.

### 3 Stage 3: Production

Each firm maximises profits by choosing the quantity of goods produced for the domestic and the foreign market. Throughout the paper we assume the parameter  $a$  to be large enough relative to trading costs  $t$  for two-way intra-industry trade to occur in equilibrium. The corresponding first-order conditions are then given by:

$$\begin{aligned}\frac{\partial \Pi}{\partial x} &= a - 2x - ey - w + s = 0, \\ \Rightarrow x &= \frac{a - w + s}{2} - \frac{ey}{2},\end{aligned}\tag{14}$$

$$\begin{aligned}\frac{\partial \Pi}{\partial x^*} &= a - 2x^* - ey^* - w + s - t = 0, \\ \Rightarrow x^* &= \frac{a - w + s - t}{2} - \frac{ey^*}{2},\end{aligned}\tag{15}$$

$$\begin{aligned}\frac{\partial \Pi^*}{\partial y} &= a - 2y - ex - w^* + s^* - t = 0, \\ \Rightarrow y &= \frac{a - w^* + s^* - t}{2} - \frac{ex}{2},\end{aligned}\tag{16}$$

$$\begin{aligned}\frac{\partial \Pi^*}{\partial y^*} &= a - 2y^* - ex^* - w^* + s^* = 0, \\ \Rightarrow y^* &= \frac{a - w^* + s^*}{2} - \frac{ex^*}{2}.\end{aligned}\tag{17}$$

Equations (14)-(17) are the output reaction functions of the respective firm given the output choices of its competitor, the firm-specific union wage, and the wage

subsidy paid by the government. Solving the system of equation yields the optimal production quantities:

$$x = \frac{(2-e)a - 2(w-s) + e(w^* - s^*) + et}{4 - e^2}, \quad (18)$$

$$x^* = \frac{(2-e)a - 2(w-s) + e(w^* - s^*) - 2t}{4 - e^2}, \quad (19)$$

$$y = \frac{(2-e)a - 2(w^* - s^*) + e(w-s) - 2t}{4 - e^2}, \quad (20)$$

$$y^* = \frac{(2-e)a - 2(w^* - s^*) + e(w-s) + et}{4 - e^2}. \quad (21)$$

Given the linear production technology, total employment of firm 1 and 2, respectively, can then be written as:

$$L = x + x^* = \frac{(2-e)(2a-t) - 4(w-s) + 2e(w^* - s^*)}{4 - e^2}, \quad (22)$$

$$L^* = y + y^* = \frac{(2-e)(2a-t) - 4(w^* - s^*) + 2e(w-s)}{4 - e^2}. \quad (23)$$

Employment is therefore decreasing in a firm's own net production costs but increasing in the production costs of its competitor.

## 4 Stage 2: Wage Bargaining

When setting the wage rate each trade union faces a trade off between wages and employment. By differentiating the unions' utility functions with respect to the wage rate, and using the expressions for employment in (22) and (23), we obtain the following first order conditions:

$$\begin{aligned} \frac{\partial \Omega}{\partial w} &= \frac{(2-e)(2a-t) - 4(w-s) + 2e(w^* - s^*)}{4 - e^2} \\ &\quad - \frac{4(w-b)}{4 - e^2} = 0, \\ \Rightarrow w &= \frac{b}{2} + \frac{(2-e)(2a-t) + 4s + 2e(w^* - s^*)}{8}, \end{aligned} \quad (24)$$

$$\begin{aligned} \frac{\partial \Omega^*}{\partial w^*} &= \frac{(2-e)(2a-t) - 4(w^* - s^*) + 2e(w-s)}{4 - e^2} \\ &\quad - \frac{4(w^* - b^*)}{4 - e^2} = 0, \\ \Rightarrow w^* &= \frac{b^*}{2} + \frac{(2-e)(2a-t) + 4s^* + 2e(w-s)}{8}. \end{aligned} \quad (25)$$

Equations (24) and (25) can be interpreted as the optimal response of each union to the wage set by the competing union and the subsidies paid by each government. Solving the system of equations yields:

$$w = \frac{8b + 2eb^* + \frac{1}{2}(4+e)(2-e)(2a-t) + (8-e^2)s - 2es^*}{16-e^2}, \quad (26)$$

$$w^* = \frac{8b^* + 2eb + \frac{1}{2}(4+e)(2-e)(2a-t) + (8-e^2)s^* - 2es}{16-e^2}. \quad (27)$$

The domestic wage rate thus increases in both the domestic and the foreign unemployment benefits as well as in the subsidy paid by the domestic government. In contrast, domestic wages decrease in the subsidy paid to the foreign firm. The latter finding is due to the fact that foreign wage subsidies, by decreasing production and thus employment of the domestic firm, also decrease the marginal benefits of higher wage demands.

## 5 Stage 1: Welfare Maximising Wage Subsidies

At this stage governments choose the welfare-maximising subsidy levels taking into account the optimal response of unions and firms abroad. Governments can either set their own (national) wage subsidies unilaterally to maximise national welfare or they can choose to cooperate and maximise joint welfare. Cooperating governments are assumed to harmonise their subsidy policies. We first analyse the case of symmetric labour markets and then allow for cross-country differences in the unemployment benefit systems.<sup>10</sup>

### 5.1 Symmetric Labour Markets

In the case of symmetric labour markets both countries pay the same level of unemployment benefits  $b = b^*$  per unemployed worker. Given the symmetry of the model, in this subsection we shall only present equations for Home. Analogous equations exist for Foreign as well.

In our model utility is derived from the consumption of the differentiated goods  $X$ ,  $Y$  and the numeraire good  $m$ . The latter in turn depends on income net off

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<sup>10</sup>Cross-country differences in unemployment benefits are not only widespread in reality, they also serve well to illustrate the general problems associated with policy harmonisation in the presence of idiosyncratic institutional features. An interesting extension could consider the case where one country has a unionised and the other a perfectly competitive labour market.

taxes and net off consumption expenditures for  $X$  and  $Y$ . Before proceeding, it is useful to reformulate the social welfare function in (12) in the following way:

$$\begin{aligned}
W &= \underbrace{1/2(x^2 + y^2) + exy}_{\text{Consumer Surplus}} + \underbrace{\Pi + (x + x^*)w + (1 - x - x^*)b}_{\text{Gross Income}} + Z \\
&\quad - \underbrace{s(x + x^*) - b(1 - x - x^*)}_{\text{Taxes}} \\
&= \underbrace{1/2(x^2 + y^2) + exy}_{\text{Consumer Surplus}} + \underbrace{\Pi + (x + x^*)w - (x + x^*)s}_{\text{Net Income}} + Z \\
&\equiv CS + \Pi + LI - GS + Z. \tag{28}
\end{aligned}$$

Welfare thus increases with consumer surplus ( $CS$ ), producer surplus ( $\Pi$ ) and labour income of the employed ( $LI$ ) but decreases with government spending on wage subsidies ( $GS$ ).

**Decomposing the Effects of Wage Subsidies.** Before calculating the optimal subsidy levels with and without policy cooperation, we consider the various effects a wage subsidy has on welfare at home and abroad. Differentiating welfare in Home with respect to the domestic subsidy  $s$  yields:

$$\frac{\partial W}{\partial s} = \frac{\partial CS}{\partial s} + \frac{\partial \Pi}{\partial s} + \frac{\partial LI}{\partial s} - \frac{\partial GS}{\partial s}. \tag{29}$$

Taking a closer look at the different elements of equation (29), we first of all see that a higher wage subsidy increases domestic consumer surplus by reducing market prices and alleviating the under-provision problem created by product market imperfections:

$$\begin{aligned}
\frac{\partial CS}{\partial s} &= x \left( \frac{\partial x}{\partial s} + e \frac{\partial y}{\partial s} \right) + y \left( \frac{\partial y}{\partial s} + e \frac{\partial x}{\partial s} \right) \\
&= \frac{(16 - 6e^2)x + (12e - 2e^3)y}{64 - 20e^2 + e^4} > 0. \tag{30}
\end{aligned}$$

Notice that the positive effect of an increase in  $s$  on consumer surplus increases with  $x$  and  $y$ . A consumer gains more from a given reduction in prices the higher her consumption level is.

Secondly, higher domestic wage subsidies increase the profit level of the domestic firm:

$$\frac{\partial \Pi}{\partial s} = \frac{4(8 - e^2)(x + x^*)}{64 - 20e^2 + e^4} > 0. \tag{31}$$

Wage subsidies drive a wedge between the wage rate and per-unit labour costs

and thus reduce marginal production costs. The firm benefits more from a given reduction in its marginal costs, when production  $x + x^*$  is large.

Thirdly, wage subsidies also increase total labour income, as given by the product of wages and employment:

$$\begin{aligned}\frac{\partial LI}{\partial s} &= (x + x^*)\frac{\partial w}{\partial s} + w\frac{\partial(x + x^*)}{\partial s} \\ &= \frac{(x + x^*)(8 - e^2)}{16 - e^2} + \frac{4w(8 - e^2)}{64 - 20e^2 + e^4} > 0.\end{aligned}\quad (32)$$

In particular, firms respond to lower labour costs by expanding production and thus employment. Moreover, higher levels of employment induce unions to demand higher wages (which somewhat dampens the positive employment effect). The effect of  $s$  on labour income increases with both the level of employment and the wage rate. The positive wage effect will apply to a larger number of workers if employment is already high. Likewise, a given increase in employment will have larger income effects if wages are high.

Taken together, wage subsidies do not only increase employment but also boost consumer surplus, profits and wages. The welfare costs for the home economy come in terms of higher government spending on wage subsidies:

$$\begin{aligned}\frac{\partial GS}{\partial s} &= s\frac{\partial(x + x^*)}{\partial s} + (x + x^*) \\ &= \frac{4s(8 - e^2)}{64 - 20e^2 + e^4} + (x + x^*) > 0.\end{aligned}\quad (33)$$

The first term on the right-hand side of equation (33) represents additional expenditures for those workers that would have been unemployed without the marginal increase in  $s$ . Moreover, the state also has to pay higher subsidies for those already employed. This effect, reflected by the second term on the right-hand side, is usually referred to as the deadweight effect of wage subsidies. The costs of further increases in  $s$  therefore depend positively on the actual level of employment. Higher government spending increases lump-sum taxes and thus reduces disposable income available for the consumption of the numeraire good.

Beside the effect a wage subsidy has on the domestic economy, it also affects welfare of the trading partner abroad. The effect of foreign wage subsidies on welfare in Home can again be broken down into four parts:

$$\frac{\partial W}{\partial s^*} = \frac{\partial CS}{\partial s^*} + \frac{\partial \Pi}{\partial s^*} + \frac{\partial LI}{\partial s^*} - \frac{\partial GS}{\partial s^*}.\quad (34)$$

Combining these four effects we identify two spill-over effects that a foreign wage subsidy has on the home economy: First, the foreign wage subsidy decreases the marginal cost of the foreign supplier and thus increases foreign production. This, in turn, reduces the price level of  $X$  and  $Y$  in Home. We therefore observe a *positive consumer surplus spill-over*:

$$\frac{\partial CS}{\partial s^*} = \frac{(24e - 4e^3)x + (32 - 12e^2)y}{64 - 20e^2 + e^4} > 0. \quad (35)$$

When product markets are linked through trade, consumer thus benefit from wage subsidies paid to the foreign firm.

Second, wage subsidies paid to the foreign firm have a *negative income effect* on Home, which is, however, only present for  $e > 0$ . In that case the two producers are competitors and national subsidy policies affect global patterns of production, of employment and of wages. In contrast, for  $e = 0$  firms are monopolists in their own market segments. Subsidies paid to the foreign firm then do not affect the production decision of the domestic firm (and vice versa). Consequently, profits, employment and wages and thus income in the home country are also not affected by foreign wage subsidies.

The income spill-over effect can be decomposed in three parts. First, a wage subsidy in Foreign shifts rents to the foreign firm and as a result profit income in Home decreases:

$$\frac{\partial \Pi}{\partial s^*} = \frac{-8e(x + x^*)}{64 - 20e^2 + e^4} < 0. \quad (36)$$

Second, a wage subsidy paid in Foreign reduces labour income in Home. This is due to both a fall in employment and in wages. By differentiating employment in Home with respect to  $s^*$ , and taking also into account the wage responses by the trade unions, we arrive at:

$$\frac{\partial(x + x^*)}{\partial s^*} = -\frac{2e}{4 - e^2} - \frac{4}{4 - e^2} \frac{\partial w}{\partial s^*} + \frac{2e}{4 - e^2} \frac{\partial w^*}{\partial s^*} \quad (37)$$

$$= -\frac{2e}{4 - e^2} \left( 1 - \frac{12 - e^2}{16 - e^2} \right) < 0. \quad (38)$$

A wage subsidy in Foreign improves the competitiveness of the foreign firm and has a direct negative effect on employment in the home country (first term on the right-hand side of 37). In unionised labour markets this effect is mitigated as unions at home and abroad adjust their wage demands accordingly. More specifically, Home's union moderates its wage demands (second term on the right-hand side of 37) while the foreign union sets a higher wage rate (third term). Overall, the



negative direct effect prevails and subsidies therefore have a negative effect on the level of employment of the trading partner. As shown, wages in Home also decrease and so does labour income:

$$\begin{aligned}\frac{\partial LI}{\partial s^*} &= (x + x^*) \frac{\partial w}{\partial s^*} + w \frac{\partial(x + x^*)}{\partial s^*} \\ &= \frac{-2e(x + x^*)}{16 - e^2} - \frac{8ew}{64 - 20e^2 + e^4} < 0.\end{aligned}\quad (39)$$

Note that the effects of foreign wage subsidies on both domestic employment and domestic wages ( $\frac{\partial(x+x^*)}{\partial s^*}$  and  $\frac{\partial w}{\partial s^*}$ ) decrease with  $e$  and are thus larger (in absolute terms) when the two goods are close substitutes in consumption.

Finally, the negative effect of foreign wage subsidies on domestic employment also implies that government expenditures on wage subsidies decline. This reduces the tax burden for consumers and hence has a positive effect on home income:

$$\frac{\partial GS}{\partial s^*} = s \frac{\partial(x + x^*)}{\partial s^*} = \frac{8es}{64 - 20e^2 + e^4} < 0.\quad (40)$$

Nevertheless, by reducing both profit and labour income, a wage subsidy in Foreign reduces *net* income in Home and thus decreases (*ceteris paribus*) the consumption level of the numeraire good.<sup>11</sup>

Summarising these policy spill-overs we arrive at:

**Proposition 1** *A foreign wage subsidy has two spill-over effects on welfare in Home. First, there is a positive effect on consumer surplus in the home country and, second, there is a negative effect on net income in Home. The negative effect on net income is only present for  $e > 0$  and operates through both a reduction in profit and in labour income.*

**Harmonisation vs. Policy Competition.** Having discussed the various effects of wage subsidies on welfare at home and abroad, we now calculate and compare the optimal subsidies with and without policy cooperation. Consider first the case of non-cooperative policy making. Each government decides independently about the level of subsidies taking the choice of the other country as given. The

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<sup>11</sup>Evaluating  $\frac{\partial LI}{\partial s^*} + \frac{\partial \Pi}{\partial s^*} - \frac{\partial GS}{\partial s^*}$  at the symmetric equilibrium  $s = s^*$ , we obtain  $\frac{-8e((6-e^2)(2a-t)-(4-(2+e)e)(b-s))}{(4-e)^2(2+e)^2(2-e)(4+e)}$ . The effect of  $s^*$  on net income in Home will be negative if  $(6-e^2)(2a-t) - (4-(2+e)e)(b-s) > 0$ . Furthermore, output of the domestic firm in the symmetric equilibrium is given by  $\frac{2(2a-t)-4(b-s)}{(4-e)(2-e)}$ . It is easily verified that nonnegative output levels then imply a negative effect of  $s^*$  on net income in Home.

first-order condition of the government in Home then reads

$$\frac{\partial W}{\partial s} = \frac{\partial CS}{\partial s} + \frac{\partial \Pi}{\partial s} + \frac{\partial LI}{\partial s} - \frac{\partial GS}{\partial s} = 0 \quad (41)$$

and the optimal non-cooperative subsidy level,  $s_{noncoop}$ , is given by

$$s_{noncoop} = b + 2a\phi - t\gamma \quad (42)$$

with  $\phi = \frac{40+6e-e^2(11+e-e^2)}{2(6+e)(4+e-e^2)} \geq \gamma = \frac{96-24e-(3-e)e^2(4-2e-e^2)}{4(2-e)(6+e)(4+e-e^2)} > 0$ . In Appendix A.1 we show that the second-order conditions for a maximum are fulfilled and that the equilibrium is stable. Equilibrium wage, price and output levels associated with (42) can be found in Appendix A.2.<sup>12</sup>

The optimal wage subsidy thus increases with  $a$  but decreases with  $t$ . An increase in  $a$  shifts the demand function for the differentiated goods outward and ceteris paribus increases production and thus employment. Even though wage subsidies are then relatively expensive, they are also more effective in raising consumer surplus and income.<sup>13</sup> Likewise, closer economic integration between the two countries (a lower  $t$ ) increases competitive pressures in the two markets and thus total production of the domestic firm (i.e.  $\frac{\partial x+x^*}{\partial t} < 0$ ). The marginal effect of wage subsidies on profit and labour income is then again larger but marginal costs of subsidising wages increase as well. In contrast to an increase in  $a$ , closer economic integration has an ambiguous effect on  $\frac{\partial CS}{\partial s}$ . While protection of the domestic market and, hence, domestic sales  $x$  decline when trade barriers are dismantled, exports into the home market  $y$  increase. Consumers then gain less from a given reduction in  $p_x$  but more

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<sup>12</sup>With just one firm being present in each market, product market imperfections are severe in our model. This is one reason for why governments have large incentive to subsidise wages. As a result, the optimal wage subsidy in (42) has the unrealistic feature of being larger than the equilibrium wage itself. When calculating the equilibrium solution, we implicitly assume that governments pay wage subsidies only for ‘productive workers’ who are hired to produce output actually sold into the market. Thus, by assumption, firms cannot hire workers and just stockpile their production (otherwise governments would not pay such high subsidies in the first place). While the fact that  $s$  is larger than  $w$  in equilibrium is clearly an undesirable model feature, it does not invalidate the message of the paper. One possible remedy to the problem is to increase the social costs of wage subsidies, e.g. by requiring them to be financed through distortionary taxes.

<sup>13</sup>Formally, we find

$$\begin{aligned} \frac{\partial CS}{\partial s \partial a} &= \frac{4(1+e)}{(4-e)^2(2+e)^2} > 0, & \frac{\partial \Pi}{\partial s \partial a} &= \frac{16(8-e^2)}{(8+2e-e^2)^2(8-2e-e^2)} > 0, \\ \frac{\partial LI}{\partial s \partial a} &= \frac{8(8-e^2)}{(4-e)^2(8+6e+e^2)} > 0, & \frac{\partial GS}{\partial s \partial a} &= \frac{4}{8+2e-e^2} > 0. \end{aligned}$$

from a reduction in  $p_y$ .<sup>14</sup>

Moreover, according to (42), the optimal wage subsidy increases one for one with domestic unemployment benefits. In fact, from a welfare perspective, unemployment benefits act as a negative wage subsidy in the model. The optimally chosen levels of output, and thus consumer surplus and profit levels, depend only on the difference between domestic subsidies and unemployment benefits (but not on their absolute levels).<sup>15</sup> Intuitively, wage subsidies decrease domestic labour costs, while unemployment benefits, by improving the bargaining position of unions, increase wages. Since wage subsidies and unemployment benefits in Foreign have an analogous effect on foreign production, and output levels are strategic substitutes, domestic production not only increases with  $s - b$  but also decreases with  $s^* - b^*$ . Moreover, total non-profit income net off lump-sum taxes  $w(x + x^*) - s(x + x^*)$ , and thus the remaining part of the welfare function, is also a function of  $s - b$ .<sup>16</sup> Unemployment benefits increase the wage rate (and thus the after-tax labour income) but decrease employment. An increase in  $s$ , in contrast, exactly pulls into the opposite direction by increasing employment but lowering the wage rate net off taxes. The domestic government then effectively chooses the optimal level of  $s - b$  taking the unemployment benefit as given.

The spill-overs described before provide a rational for policy coordination. Since governments do not take costs and benefits for the foreign country into consideration, the non-cooperative solution in (42) is generally not optimal from a global point of view. We thus compare the non-cooperative solution to a cooperative equilibrium. Motivated by the literature on tax competition, we first assume that governments

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<sup>14</sup>Formally, we find

$$\begin{aligned} \frac{\partial \Pi}{\partial s \partial t} &= \frac{-8(8 - e^2)}{(8 + 2e - e^2)^2(8 - 2e - e^2)} < 0, & \frac{\partial LI}{\partial s \partial t} &= \frac{-4(8 - e^2)}{(4 - e)^2(8 + 6e + e^2)} < 0, \\ \frac{\partial GS}{\partial s \partial t} &= \frac{-2}{8 + 2e - e^2} > 0, & \frac{\partial CS}{\partial s \partial t} &= \frac{32 - 40e - e^2(20 - 6e - (3 - e)e^2)}{(4 - e)^2(4 + e)(4 - e^2)^2}, \end{aligned}$$

where the last expression is positive (negative) for a sufficiently small (large)  $e$ .

<sup>15</sup>By plugging the wages set by the unions into the corresponding output expressions for the domestic firm, we obtain

$$\begin{aligned} x &= \frac{4(4 + e)(2 - e)a + (16 + 20e - e^3)t + (32 - 4e^2)(s - b) - 8e(s^* - b^*)}{2(64 - 20e^2 + e^4)}, \\ x^* &= \frac{4(4 + e)(2 - e)a - (4 + e)(12 - e^2)t + (32 - 4e^2)(s - b) - 8e(s^* - b^*)}{2(64 - 20e^2 + e^4)}. \end{aligned}$$

Analogous expressions exist for the firm located in Foreign.

<sup>16</sup>Using equation (26),  $w - s$  can be written as  $\frac{(4+e)(2-e)(2a-t)-16(s-b)-4e(s^*-b^*)}{2(16-e^2)}$ .

agree to harmonise the level of wage subsidies across countries. This approach seems natural if countries are fully symmetric. It will, however, entail costs if countries differ in terms of their labour market institutions. As an extension, we thus also consider an optimal cooperation policy (see Section 6.1).

Cooperating governments set the common subsidy level  $s$  so as to maximise aggregate welfare  $W + W^* = (CS + CS^*) + (\Pi + \Pi^*) + (LI + LI^*) - (GS - GS^*) + (Z + Z^*)$ . Solving the resulting first-order condition then yields the common welfare-maximising subsidy level under policy harmonisation

$$s_{coop1} = b + \frac{(6 - e^2)(2a - t)}{4(1 + e)}, \quad (43)$$

where the subscript *coop1* indicates that we consider cooperation with just one instrument (namely the common harmonised wage subsidy) at hand. The associated equilibrium wages, prices and output levels can again be found in the Appendix.

The difference between the optimal subsidy under policy cooperation and non-cooperation is given by

$$s_{coop1} - s_{noncoop} = \frac{(4 - e)^2(1 - e)(2 + e)^2((2 - e)2a - 3t)}{4(2 - e)(1 + e)(6 + e)(4 + e - e^2)} \quad (44)$$

and the welfare gain from policy harmonisation can be written as:

$$\begin{aligned} & (W_{coop1} + W_{coop1}^*) - (W_{noncoop} + W_{noncoop}^*) = \\ & \frac{(4 - e)^2(1 - e)^2(2 + e)^2((2 - e)2a - 3t)^2}{2(2 - e)^2(1 + e)(6 + e)^2(4 + e - e^2)^2}. \end{aligned} \quad (45)$$

By inspecting the differences in (44) and (45), we arrive at

**Proposition 2** *If governments maximise social welfare and countries are symmetric, the optimal subsidy level under policy harmonisation will be strictly higher than under non-cooperation for  $e = [0, 1[$  and identical for  $e = 1$ . Furthermore, the difference between the subsidy level with and without policy cooperation and the welfare gain from cooperation are both strictly decreasing in  $e$ .*

The proof of Proposition 2 is relegated to the Appendix.

Intuitively, Proposition 2 can be explained as follows: While the positive spill-over effect on consumer surplus calls for a higher cooperative subsidy, the negative income spill-over pulls into the opposite direction. As long as the two goods are not fully homogeneous, the positive spill-over effect on consumer surplus dominates

and cooperating governments choose a higher subsidy level than non-cooperating governments do. In fact, the gap between the subsidy levels in the cooperative and the non-cooperative equilibrium increases with the degree of product differentiation. To grasp the intuition behind the result, it is useful to reconsider the extreme case of  $e = 0$ . Recall that the two firms are then monopolies in their market segments and a wage subsidy paid to one firm does not affect production of the other. Employment and wages in Home are also not affected by wage subsidies in Foreign. The negative income spill-over effect then ceases to exist and cooperating governments are left with the positive spill-over effect on consumer surplus:

$$\begin{aligned} \left. \frac{\partial W}{\partial s^*} \right|_{e=0} &= \left. \frac{\partial CS}{\partial s^*} \right|_{e=0} + \left. \frac{\partial \Pi}{\partial s^*} \right|_{e=0} + \left. \frac{\partial LI}{\partial s^*} \right|_{e=0} - \left. \frac{\partial GS}{\partial s^*} \right|_{e=0} \\ &= \left. \frac{\partial CS}{\partial s^*} \right|_{e=0} > 0. \end{aligned} \tag{46}$$

Consequently, cooperating governments have an incentive to agree on a relatively large wage subsidy (compared to the non-cooperative solution). Naturally, large differences between  $s_{coop1}$  and  $s_{noncoop}$  also imply large welfare gains from policy harmonisation.

## 5.2 Asymmetric Labour Market Institutions

Differences in national institutions (or different preferences about institutional features) are often seen as the main culprit why cooperation, though theoretically appealing, rarely works in practice. In this section we thus allow for asymmetric labour market institutions, namely for cross-country differences in the level of unemployment benefits, and discuss how these asymmetries alter our results derived before. Without loss of generality, we assume that  $b > b^*$ . This may, for instance, replicate institutional differences in the unemployment benefits system across EU member states such as Sweden and the UK.

We find that policy harmonisation will face two problems if countries differ in the generosity of their unemployment benefits: First, policy harmonisation reduces the capability of governments to adopt their policies to country-specific institutions. And second, even if policy harmonisation still increases *aggregate* welfare, it can run into fierce political opposition because benefits and costs are unevenly distributed between cooperating countries. Surprisingly, a harmonisation of wage subsidies, in fact, increases cross-country differences in welfare levels.

We first derive the non-cooperative equilibrium. Governments set wage subsidies to maximise national welfare taking the subsidy choice of the trading partner as

given. To simplify expressions, we choose a specific value of  $e$  and assume that the two products are completely unrelated (i.e.  $e = 0$ ). The results presented in the following are, in a qualitative sense, not affected by the specific choice of  $e$  as long as  $e$  is smaller than one (and thus potential gains from cooperation exist).<sup>17</sup> For  $e = 0$  the non-cooperative subsidy levels are given by:

$$s_{noncoop} = b + \frac{5}{3}a - \frac{1}{2}t, \quad s_{noncoop}^* = b^* + \frac{5}{3}a - \frac{1}{2}t. \quad (47)$$

The optimal subsidy set by each government thus increases with the respective domestic level of unemployment benefits but is unrelated to the benefits paid by the trading partner. As in the case of symmetric labour markets, subsidies increase in  $a$  but decrease in  $t$ .

The one-to-one relation between subsidies and unemployment benefits imply that both countries choose the same difference  $s - b = s^* - b^*$ . Welfare in turn depends only on this difference but not on the absolute levels of  $s$  and  $b$ . Therefore, the non-cooperative equilibrium levels of welfare are independent of the country-specific and exogenously given unemployment benefits:

$$W_{noncoop} - Z = W_{noncoop}^* - Z^* = \frac{8a^2}{9} - \frac{5at}{6} + \frac{3t^2}{8}. \quad (48)$$

Wage subsidies allow each government to fully offset the welfare effects of unemployment benefits. Apart from the exogenous endowment of non-labour assets, welfare levels in the two countries are thus exactly identical in the non-cooperative equilibrium. Clearly, this result should not be taken too literally since it hinges on a number of assumptions, and in particular on the capability of governments to raise non-distortionary (lump-sum) taxes. In fact, for  $b > b^*$  Home is a high-wage, high-tax country while both taxes and wages are relatively low in Foreign. The results in (47) and (48) mainly serve as a benchmark scenario, to which we now compare welfare under policy harmonisation.

If governments agree to harmonise their labour market policies, they choose a common subsidy level  $s = s^*$  to maximise joint welfare. The resulting optimal subsidy level for  $e = 0$  reads:

$$s_{coop1} = \frac{b + b^*}{2} + \frac{3(2a - t)}{2}. \quad (49)$$

When comparing subsidy levels with and without policy coordination, two main differences are striking. First, a harmonisation of wage subsidies has the distinctive

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<sup>17</sup>For  $e = 1$  the cooperative and the non-cooperative solution are again identical in welfare terms.

disadvantage that labour market policies can no longer be targeted to the benefit system of each country. In fact, the common subsidy depends on the cross-country average of the two unemployment benefit levels. Second, with policy coordination the two governments account for spill-over effects of wage subsidies. Since the overall spill-over effect on welfare is positive, the common subsidy level under cooperation is strictly higher than the *average* of the two non-cooperative subsidy levels.<sup>18</sup> It is the interplay of these two forces - the cost of not being able to tailor the wage subsidy to the characteristics of the domestic labour market and the general benefit of internalising the spill-over effects of wage subsidies - that is at the heart of any cost and benefit analysis of policy harmonisation in the presence of asymmetric labour markets.

If cooperating governments choose a common subsidy level, wage subsidies paid in one country not only reflect domestic but also foreign labour market institutions. Therefore, cross-country differences in the unemployment benefit systems are no longer inconsequential for domestic welfare. In equilibrium, welfare levels are given by:

$$W_{coop1} - Z = a^2 + \frac{a(b - b^* - 4t)}{4} - \frac{(b - b^* - 2t)(b - b^* + 14t)}{64}, \quad (50)$$

$$W_{coop1}^* - Z^* = a^2 + \frac{a(b^* - b - 4t)}{4} - \frac{(b^* - b - 2t)(b^* - b + 14t)}{64}. \quad (51)$$

Inspecting the expressions (50) and (51) and comparing them to the results derived for the case of non-cooperating governments, we arrive at

**Proposition 3** *i. Policy harmonisation increases (decreases) joint welfare of the two countries if and only if  $b - b^* < (>) \frac{2(4a-3t)}{3}$ . Furthermore, the aggregate welfare gain from cooperation is strictly decreasing in  $b - b^*$ .*

*ii. Policy harmonisation increases (decreases) welfare in Home if and only if  $b - b^* < (>) \frac{2\lambda(4a-3t)}{3}$  with  $\lambda = \sqrt{10} + 3 > 1$ . For sufficiently small (large) differences between  $b$  and  $b^*$ ,  $W_{coop1} - W_{noncoop}$  is increasing (decreasing) in  $b - b^*$ .*

*iii. Policy harmonisation increases (decreases) welfare in Foreign if and only if  $b - b^* < (>) \frac{2\lambda^*(4a-3t)}{3}$  with  $\lambda^* = \sqrt{10} - 3 < 1$ . Moreover,  $W_{coop1}^* - W_{noncoop}^*$  is strictly decreasing in  $b - b^*$ .*

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<sup>18</sup>Subtracting  $1/2(s_{noncoop} + s_{noncoop}^*)$  from  $s_{coop1}$  gives  $4/3a - t$ . Given our assumption of positive intra-industry trade and the fact that  $x$  and  $y$  equal  $(4a - 3t)/6 > 0$  in the non-cooperative equilibrium, the common subsidy level under cooperation is higher than the average of the two non-cooperative subsidy levels.

- iv. *With policy coordination welfare net of the exogenous endowment of non-labor assets is strictly larger in Home than in Foreign, i.e.  $W_{coop1} - Z > W_{coop1}^* - Z^*$ .*

The proof of Proposition 3 is relegated to the Appendix.

The proposition demonstrates that even with cross-country differences in labour market institutions harmonising subsidies can raise joint welfare of the two countries - as long as the institutions in place are not too different. The aggregate welfare gain successively decreases and eventually turns negative as cross-country differences in labour market institutions become more important. However, this may not be the case for each country on its own. In fact, cross-country differences in the benefit systems strictly diminish the welfare gain from policy harmonisation for Foreign (that pays the relatively low unemployment benefit level of  $b^* < b$ ). In contrast, for Home the gain from coordination even increases with  $b - b^*$  provided that the difference does not exceed a certain threshold level. Moreover, in the cooperative equilibrium welfare in Home, the country with a more generous unemployment benefit system, strictly exceeds welfare in Foreign. Harmonising wage subsidies therefore does not equalise welfare across countries but even drives a wedge between welfare levels in Home and in Foreign.

To clarify these results, consider the country-specific differences between the subsidy paid with and without policy cooperation:

$$s_{coop1} - s_{noncoop} = \frac{4a - 3t}{3} - \frac{b - b^*}{2}, \quad (52)$$

$$s_{coop1}^* - s_{noncoop}^* = \frac{4a - 3t}{3} + \frac{b - b^*}{2}. \quad (53)$$

If countries do not engage in policy competition but choose a common subsidy level, they will target the average unemployment benefit  $(b + b^*)/2$  rather than their country-specific benefit levels  $b$  and  $b^*$ . The average is too low relative to the generous benefits in Home but too high relative to the meager benefits in Foreign (i.e.  $b > (b + b^*)/2 > b^*$ ). It then follows that an increase in  $b - b^*$  decreases  $s_{coop1} - s_{noncoop}$  but increases  $s_{coop1}^* - s_{noncoop}^*$ . Since on average the common subsidy level is higher than the non-cooperative choices, larger cross-country differences push the harmonised subsidy closer to  $s_{noncoop}$  (as long as  $b - b^*$  is not getting too large) but further away from  $s_{noncoop}^*$ . As the non-cooperative subsidy levels represent the best response of a country to the subsidy set by its trading partner, the gain from policy harmonisation then increases in Home but decreases in Foreign. Moreover, given the positive spill-over effect associated with wage subsidies, Home does not



only benefit from  $s_{coop1}$  approaching  $s_{noncoop}$  but also from the increase in  $s_{coop1}^*$  relative to  $s_{noncoop}^*$ . Only for large differences between  $b$  and  $b^*$ , when  $s_{coop1}$  already falls short of  $s_{noncoop}$ , further increases in  $b - b^*$  push  $s_{coop1}$  away from  $s_{noncoop}$  and eventually decrease Home's benefits from policy harmonisation.

Foreign, in contrast, is punished twice by increases in  $b - b^*$ . Not only does its own wage subsidy diverges more and more from  $s_{noncoop}^*$ , it also benefits less and less from Home paying a higher subsidy under harmonisation than under non-cooperation. For large  $b - b^*$  Foreign even suffers from  $s_{coop1}$  falling short of  $s_{noncoop}$ .<sup>19</sup>

The differential effect on welfare in Home and Foreign induces a potential enforceability problem associated with policy cooperation under labour market asymmetries. According to Proposition 3 there exists a range of parameter values,  $\frac{2\lambda(4a-3t)}{3} < b - b^* < \frac{2(4a-3t)}{3}$ , for which policy harmonisation increases joint welfare of the two countries but decreases welfare in Foreign. The low-benefit country then has an incentive to blockade any policy initiative to harmonise labour market policies. Our analysis thus provides a rationale for why labour market policies are still mostly in national hands even though policy harmonisation might be beneficial from a bird's eye perspective.

## 6 Extensions

In this subsection, we consider two extensions to our benchmark model. We first study an optimal cooperation policy that allows cooperating governments to choose country-specific subsidy levels. We then analyse how our main findings would change if governments targeted employment rather than welfare.

### 6.1 Optimal Cooperative Wage Subsidies

So far, we have interpreted policy cooperation as a harmonisation of wage subsidies. In fact, the public debate about international policy coordination often focuses on the question of whether national policies should be leveled across national borders. However, policy harmonisation reduces the capability of governments to adapt their policies to country-specific institutions as governments are left with only one policy instrument at hand. An optimal cooperative policy, in contrast, allows subsidy levels to differ across countries. With asymmetric labour market institutions, aggregate

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<sup>19</sup>While an increase in  $b - b^*$  diminishes the aggregate gains from cooperation for Foreign, firm profits are positively affected. Foreign firms benefit both from relatively high wage subsidies (compared to the low unemployment benefits) and the comparably low subsidies paid to their competitors.

welfare must be higher if policy makers have two rather than one instruments at their disposition. However, as we illustrate in the following, the high-benefit country might nevertheless prefer a harmonisation policy.

Optimal policy cooperation between Home and Foreign requires that country-specific subsidy levels  $s$  and  $s^*$  are chosen to maximise aggregate welfare  $W+W^*$ . To simplify expressions - and to facilitate comparison with the results derived in Section 5.2 - we again focus on the case of unrelated products (i.e.  $e = 0$ ). We also continue to assume that Home pays higher unemployment benefits than Foreign i.e.  $b > b^*$ . Solving the first-order conditions yields the optimal cooperative subsidy levels (marked with the subscript *coop2* for a cooperation strategy with two instruments):<sup>20</sup>

$$s_{coop2} = b + \frac{3(2a - t)}{2}, \quad s^*_{coop2} = b^* + \frac{3(2a - t)}{2}. \quad (54)$$

The optimal cooperative solution combines the benefits of both policy competition and subsidy harmonisation. On the one hand, with two instruments at hand governments target country-specific institutions (as they do under policy competition) and national wage subsidies depend on local unemployment benefits only. On the other hand, cooperating governments account for the spill-over effects of wage subsidies. In fact, the average of  $s_{coop2}$  and  $s^*_{coop2}$  exactly equals the optimal harmonised subsidy level  $s_{coop1}$ .

The optimal subsidies in (54) increase one to one with national unemployment benefits. Therefore, the difference between  $s$  and  $b$  is identical in Home and Foreign and welfare levels coincide as well:

$$W_{coop2} - Z = W^*_{coop2} - Z^* = a^2 - at + \frac{7t^2}{16}. \quad (55)$$

Policy cooperation with two instruments at hand always increases *aggregate* welfare  $W + W^*$  relative to both the equilibrium with policy competition and policy harmonisation. Since welfare levels in Home and Foreign are identical, as they are under policy competition, it furthermore follows that each country on its own also prefers optimal policy cooperation to policy competition. The high-benefit country, however, might still lobby for policy harmonisation, as we demonstrate in

**Proposition 4** *i. Optimal policy cooperation decreases (increases) welfare in Home relative to the equilibrium under policy harmonisation iff  $b - b^* < (>)$   $4(4a - 3t)$ .*

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<sup>20</sup>The second-order condition for a maximum are fulfilled as  $\frac{\partial^2(W+W^*)}{\partial^2 s} = \frac{\partial^2(W+W^*)}{\partial^2 s^*} = -\frac{1}{8}$ .

*ii. Optimal policy cooperation increases welfare in Foreign relative to the equilibrium under policy harmonisation.*

The proof of Proposition 4 is relegated to the Appendix.

Even though policy-makers give up a policy instrument by harmonising subsidy levels, Home will be best off with policy harmonisation if cross-country differences are sufficiently small. Foreign, in contrast, is always better off when cooperating countries do not coordinate on a common subsidy but choose country-specific subsidy levels.

The reason is as follows. If countries have to choose a common subsidy level, they will target the average unemployment benefit  $(b + b^*)/2$ . The harmonised subsidy level is thus lower in the high-benefit country but higher in the low-benefit country (relative to the optimal cooperative and country-specific subsidies). Positive spill-over effects imply that Home benefits from higher subsidy levels paid in Foreign. In addition, for sufficiently small differences in the unemployment benefit systems, the difference between  $s_{coop1}$  and  $s_{noncoop}$  is smaller (in absolute terms) than the difference between  $s_{coop2}$  and  $s_{noncoop}$ . Subtracting the two terms yields:

$$|s_{coop2} - s_{noncoop}| - |s_{coop1} - s_{noncoop}| = \left| \frac{4a - 3t}{3} \right| - \left| \frac{4a - 3t}{3} - \frac{b - b^*}{2} \right|. \quad (56)$$

For  $b - b^* < 4(4a - 3t)/3$ , the harmonised subsidy  $s_{coop1}$  is closer to Home's optimal unilateral choice  $s_{noncoop}$  than is the optimal cooperative subsidy  $s_{coop2}$ . Since the non-cooperative subsidy represents the best response of a country to the subsidy set by its trading partner, this is again in the interest of Home.

From the perspective of Foreign, in contrast, policy harmonisation only carries disadvantages relative to the optimal cooperative policy. Not only does Home pay relatively low wage subsidies. Policy harmonisation also pushes the subsidy level further away from Foreign's non-cooperative choice than optimal policy coordination does:

$$\begin{aligned} |s_{coop2}^* - s_{noncoop}^*| - |s_{coop1}^* - s_{noncoop}^*| &= \left| \frac{4a - 3t}{3} \right| - \left| \frac{4a - 3t}{3} + \frac{b - b^*}{2} \right| \\ &= -\frac{b - b^*}{2} < 0. \end{aligned} \quad (57)$$

It then follows that Foreign strictly prefers optimal policy cooperation over policy harmonisation.

To sum up: The optimal cooperative policy accounts for the spill-over effects

of national wage subsidies while allowing policy-makers to target country-specific labour market institutions. Aggregate welfare thus increases unambiguously compared to both policy competition and policy harmonisation. But even if policy cooperation could indeed be designed optimally in the real world, countries can still disagree about which policy to implement. In fact, the high-benefit country may prefer the (globally suboptimal) harmonisation policy that chooses a common subsidy level for both countries.

## 6.2 Optimal Wage Subsidies under an Employment Target

Up to this point, we have analysed welfare maximising wage subsidies. Even though a welfare target is widely used in academic policy analysis, governments may in reality pursue different policy goals. In fact, the prime motive for subsidising wages is usually to stimulate employment (while adhering to some budget constraint). In this subsection, we therefore analyse a political reform where governments maximise the value of employment ( $VE$ ) and compare the results to those derived for a welfare maximising policy.

Governments are assumed to value each additional job at the difference between the unemployment benefit saved by the state and the subsidy paid:<sup>21</sup>

$$VE = (b - s)L, \quad VE^* = (b^* - s^*)L^*. \quad (58)$$

Government then set wage subsidies such that, at the margin, the cost of creating an additional job equals the unemployment benefit saved by the state. Note that our employment target function is particularly conservative as governments only subsidise employment if the additionally created job can be financed by the savings in unemployment benefits for this otherwise unemployed new employee. Consequently, governments implicitly minimise their overall expenditures on unemployment benefits and subsidies.

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<sup>21</sup>Clearly, this assumption is to a certain degree arbitrary. However, the main purpose of the subsection is not to give an accurate description of governments' behaviour. We rather want to illustrate that results can depend on the specific policy target chosen. One could, of course, envisage different employment target functions. For instance, the government could value an additional job by its market wage rather than by the unemployment benefit saved. Non-standard target functions have already been used in the literature on wage and hiring subsidies. For example, Fredriksson and Holmlund (2006) maximise social welfare of workers subject to non-workers receiving at least a minimum amount of utility. Orszag and Snower (2003) employ a welfare objective such that the number of (long-term) unemployed is minimised and welfare of low-wage earners is maximised.

Consider first the effect of a domestic wage subsidy on the domestic government target. For the sake of brevity we focus on the case of symmetric countries only.<sup>22</sup> Given the symmetry of the model, we again restrict our analysis to the home country. Differentiating the employment target with respect to the subsidy level yields:

$$\frac{\partial VE}{\partial s} = \frac{\partial L}{\partial s}(b - s) - L, \quad (59)$$

where  $L = (x + x^*)$  and  $\frac{\partial L}{\partial s} = \frac{4(8-e^2)}{64-20e^2+e^4} > 0$ .

The government faces the same marginal costs as under welfare targeting. In particular, a marginal increase in the subsidy level boosts government's expenditure on wage subsidies. Some of that additional government expenditure is paid for new workers joining the labour force ( $\frac{\partial L}{\partial s}s$ ), the rest is again a *deadweight loss* as the higher subsidy also has to be paid for those workers already in employment ( $L$ ). But higher wage subsidies, by reducing unemployment, also lower government expenditures on unemployment benefits and thus increase the value of employment ( $\frac{\partial L}{\partial s}b$ ). Importantly, a government that targets employment rather than welfare does only account for a subset of the policy effects discussed in Section 5.1. In particular, governments ignore the positive effect of wage subsidies on consumer surplus and gross income.

In addition to the effect of the domestic subsidy on the home economy, a wage subsidy also affects (the value of) employment abroad. Differentiating the domestic target function with respect to the level of foreign wage subsidies yields:

$$\frac{\partial VE}{\partial s^*} = \frac{\partial L}{\partial s^*}(b - s), \quad (60)$$

where

$$\frac{\partial L}{\partial s^*} = -\frac{2e}{4-e^2} \left( 1 - \frac{12-e^2}{16-e^2} \right) < 0. \quad (61)$$

As discussed before, the wage subsidy abroad has a negative spill-over effect on employment in the home economy. Under an employment target the marginal effect on employment is valued at  $b - s$ , the net cost to the state of any worker additionally unemployed. Neither the positive consumer surplus spill-over nor the negative income spill-over effect appears in (60).

In the following, we derive the optimal wage subsidy with and without policy coordination. First, let us assume that governments choose the optimal subsidy

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<sup>22</sup>A detailed discussion for the case of asymmetric institutions can be obtained from the authors upon request

unilaterally by maximising their domestic target functions. The optimal subsidy can then be written as:

$$s_{noncoop}^{VE} = s_{noncoop}^{VE*} = b - \frac{(8 - 2e - e^2)(2a - t)}{4(8 - e - e^2)}, \quad (62)$$

where we use the superscript  $VE$  to distinguish the solution from the welfare-maximising choice. We show in Appendix D.1 that the second-order conditions for a maximum are fulfilled and that the equilibrium is stable.

The optimal subsidy in equilibrium is thus increasing in  $b$  and  $t$  but decreasing in  $a$ . Higher unemployment benefits not only aggravate the unemployment problem by increasing the reservation wage of workers but also increase the per-capita costs of the unemployed to the state. In response, governments set higher subsidy levels. Lower trading costs, in contrast, intensify competition in the product market and thus increase employment. The same is true for increases in  $a$  that shift out the demand function for the differentiated goods  $X$  and  $Y$ . Additional wage subsidies then face high deadweight costs and governments choose lower wage subsidies. The qualitative effects of changes in  $a$  and  $t$  on the wage subsidy chosen by the government are thus exactly reversed compared to the welfare maximising solution. Finally, higher degrees of product differentiation decrease the optimal subsidy.<sup>23</sup> A decrease in  $e$  not only increases market demand and employment (and thus the costs of wage subsidies). It also reduces the negative employment spill-over effect on the trading partner and renders a wage subsidy less efficient in increasing domestic employment.

Given the negative employment spill-over effect associated with wage subsidies, there is room for policy coordination. With symmetric labour markets governments do not lose from having only one instrument at hand and we thus consider only the case of policy harmonisation. If governments set a common subsidy level to maximise their joint policy target  $VE + VE^* = (b - s)L + (b - s)L^*$ , the corresponding first-order condition will read:

$$\frac{\partial VE + VE^*}{\partial s} = \frac{\partial(L + L^*)}{\partial s}(b - s) - (L + L^*) = 0. \quad (63)$$

Under policy harmonisation the wage subsidy is chosen such that, at the margin, the cost for creating an additional job *in the union of both countries* equals the unemployment benefit saved by the governments. Such a policy then minimises the combined expenditures of the two states for unemployment benefits and wage

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<sup>23</sup>Formally, we find  $\frac{\partial s_{noncoop}^{VE}}{\partial e} = \frac{(8+e^2)(2a-t)}{4(8-e-e^2)^2} > 0$ .

subsidies.

Solving the maximisation problem yields the following optimal subsidy level under policy cooperation:

$$s_{coop1}^{VE} = b - \frac{2a - t}{4}. \quad (64)$$

While the wage subsidy under cooperation still increases with  $b$  and decreases with  $2a - t$ , it is independent of  $e$ . If goods are close substitutes, the negative employment spill-over effect will be high and thus the effect of an increase in  $s$  on  $L + L^*$  comparably low.<sup>24</sup> This effect balances the higher incentives for subsidising wages that result from the comparably low employment level associated with a low level of product differentiation.

Given that wage subsidies have a *negative* employment effect on the trading partner, the subsidy under policy harmonisation is lower than under policy competition (for  $e > 0$ ):

$$s_{coop1}^{VE} - s_{noncoop}^{VE} = \frac{-e(2a - t)}{4(8 - e - e^2)} < 0. \quad (65)$$

Cooperation internalises the negative employment spill-over effect and increases the equilibrium value of the policy target function in both countries.

Both under policy coordination and non-coordination, the welfare maximising government analysed in the previous section chooses a higher wage subsidy than a government pursuing an employment target. If governments target the value of employment, they will ignore the positive effect of wage subsidies on consumer surplus and gross income. Cooperation then decreases rather than increases the subsidy level. Our analysis therefore illustrates that the effects of international policy coordination may depend strongly on the policy goals pursued by national governments.

We can now summarise the main findings of this section in

- Proposition 5** *1. If countries are symmetric and governments maximise employment until the cost of creating a new job - at the margin - equals the unemployment benefit the government has to pay otherwise, wage subsidies create a negative employment spill-over effect. This effect decreases in the level of product differentiation (i.e. increases with  $e$ ). The optimal subsidy level under policy harmonisation is then strictly lower than under policy competition for  $e \in ]0, 1]$  and identical for  $e = 0$ .*
- 2. The welfare maximising subsidy levels are always higher than the subsidy levels governments choose under the employment target. More specifically, we find*

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<sup>24</sup>  $\frac{\partial(L+L^*)}{\partial s \partial e} = \frac{-16(1-e)}{(4-e)^2(2+e)^2} < 0$  for  $0 \leq e < 1$ .

$$s_{coop1} > s_{noncoop} > s_{noncoop}^{VE} > s_{coop1}^{VE}.$$

## 7 Conclusion

Wage subsidies are an increasingly popular policy tool to foster labour demand and fight unemployment without creating working poverty. When product markets are global, however, such labour market policies entail important spill-over effects, a fact widely ignored by the academic literature. In this paper, we analyse the effects of wage subsidies in an international duopoly model with unionised labour markets and discuss the costs and benefits from international policy coordination both for the case of symmetric and asymmetric labour market institutions.

We find that wage subsidies paid in one country have several effects - both for the domestic country but also for the trading partner. First, wage subsidies affect the global patterns of wages and employment, increasing labour income at home while decreasing it abroad. Second, wage subsidies increase profits of the domestic firm at the expense of the foreign competitor. Third, consumers at home and abroad benefit from wages subsidies as they reduce marginal production cost and thus consumer prices. The relative strength of these effects depends crucially on the degree of product differentiation.

Given the spill-over effects identified in our analysis, there is room for coordinating labour market policies internationally. In fact, coordinative action has been discussed in the policy arena, especially within the European Union, but little progress has been made in its implementation. We show that policy harmonisation can run into severe problems when national labour market institutions differ, as they do in practice. Not only does harmonisation reduce the capability of governments to adopt their policies to country-specific institutions. The benefits and costs are also unevenly distributed between cooperating countries. International policy harmonisation may then be difficult to enforce politically, even if it increases *aggregate* welfare. Thus, in practice, the harmonisation of labour market policies seems realistic only for countries with sufficiently similar labour market institutions, a result that also may help to explain the slow speed at which coordination within the European Union has progressed so far.

An optimal cooperative policy that allows subsidy levels to differ across countries does not restrict the capability of cooperating governments to account for country-specific institutions. It thus overcomes the key problem associated with policy harmonisation. Though theoretically appealing, an optimal cooperative policy might be very difficult to implement in reality since it requires cross-country coordination



on more than just one instrument. And even if policy cooperation could indeed be designed optimally, countries can still disagree about which cooperation policy to implement. In fact, our analysis suggests that a subset of countries could lobby for policy harmonisation although a common subsidy does not maximise overall welfare.

Our analysis lends itself to a number of extensions. In particular, we have abstracted from labour and capital mobility. While in the past labour mobility within the European Union, a prime candidate for policy coordination, has indeed been rather low, this may change as the European unification process proceeds. Since labour market policies could arguably also affect migration decisions, a model with mobile workers may detect additional spill-overs not captured in our simple framework. Likewise, labour market policies might also affect the location decision of firms, a fact also not captured in our model setup. Finally, wage subsidies are generally regarded as a mean to fight unemployment among low-skilled workers. Hence, allowing for heterogeneous labour could enrich the predictions of the model. We consider these extensions as promising paths for future research.

## References

- Brander, J. A., 1981. Intra-industry trade in identical commodities. *Journal of International Economics* 11 (1), 1–14.
- Brander, J. A., Spencer, B. J., 1984. Trade warfare: Tariffs and cartels. *Journal of International Economics* 16 (3-4), 227 – 242.
- Brander, J. A., Spencer, B. J., 1985. Export subsidies and international market share rivalry. *Journal of International Economics* 18 (1-2), 83 – 100.
- European Union, 1997. The treaty of Amsterdam. *Official Journal (C340)* 1; 37 I.L.M. 56.
- Franzese, Robert J., J., Hays, J. C., 2006. Strategic interaction among EU governments in active labor market policy-making: Subsidiarity and policy coordination under the European Employment Strategy. *European Union Politics* 7 (2), 167–189.
- Fredriksson, P., Holmlund, B., 2006. Optimal unemployment insurance design: Time limits, monitoring, or workfare? *International Tax and Public Finance* 13 (5), 565–585.
- Haufler, A., Pflüger, M., 2007. International oligopoly and the taxation of commerce with revenue-constrained governments. *Economica* 74 (295), 451–473.
- Haufler, A., Schjelderup, G., Stähler, F., 2005. Barriers to trade and imperfect competition: The choice of commodity tax base. *International Tax and Public Finance* 12 (3), 281–300.
- Krueger, A. B., Mueller, A., 2010. Job search and unemployment insurance: New evidence from time use data. *Journal of Public Economics* 94 (3-4), 298–307.
- Layard, R., Nickel, S., Jackman, R., 2005. *Unemployment*. Oxford University Press.
- Moriconi, S., Sato, Y., August 2009. International commodity taxation in the presence of unemployment. *Journal of Public Economics* 93 (7-8), 939–949.
- Naylor, R., 1998. International trade and economic integration when labour markets are generally unionised. *European Economic Review* 42 (7), 1251–1267.
- Neary, P., 2002. International trade in general oligopolistic equilibrium. University College Dublin, mimeo, revised 2009.

- Neary, P., 2003. Globalisation and market structure. *Journal of the European Economic Association* 1 (2-3), 245–271.
- Orszag, J. M., Snower, D. J., 2003. Designing employment subsidies. *Labour Economics* 10 (5), 557–572.
- Saint-Paul, G., 2000. *The political economy of labour market institutions*. Oxford University Press.
- Zaiceva, A., Zimmermann, K. F., 2008. Scale, diversity, and determinants of labour migration in europe. IZA Discussion Papers 3595, Institute for the Study of Labor (IZA).

## A Appendix to Section 5.1

### A.1 Reaction Functions and Second-Order Conditions

If governments choose the level of wage subsidies non-cooperatively, the reaction function  $s(s^*)$  for Home will read:

$$\begin{aligned}
 s(s^*) = & \frac{(4+e)(2-e)(6+e)(4+(1-e)e)b}{192-84e^2+7e^4} - \frac{e(32-8e^2+e^4)s^*}{192-84e^2+7e^4} \\
 & + \frac{(4+e)(2-e)(40+6e-e^2(11+(1-e)e))a}{192-84e^2+7e^4} \\
 & - \frac{(384-(2-e)e^2(36+4e-e^2(5+e)))t}{4(192-84e^2+7e^4)}. \tag{66}
 \end{aligned}$$

The system of reaction functions is stable if  $|\frac{\partial s}{\partial s^*}| = \frac{e(32-8e^2+e^4)s^*}{192-84e^2+7e^4} < 1$ . This condition is fulfilled for the relevant parameter range of  $e$  (i.e. for  $0 \leq e \leq 1$ ). Moreover, the second-order condition is given by:

$$\frac{\partial^2 W}{\partial^2 s} = \frac{-4(192-84e^2+7e^4)}{(64-20e^2+e^4)^2} < 0. \tag{67}$$

If, in contrast, the two governments agree to harmonise their wage subsidies, the second-order condition will be given by:

$$\frac{\partial^2 (W + W^*)}{\partial^2 s} = \frac{-16(1+e)}{(4-e)^2(2+e)^2} < 0. \tag{68}$$

### A.2 Output, Prices and Wages in Equilibrium

$$\begin{aligned}
 x_{noncoop} = y_{noncoop}^* &= \frac{(2-e)(8-e^2)2a + (10e - (2+e)e^2)t}{(6+e)(2-e)(4+e-e^2)}, \\
 x_{noncoop}^* = y_{noncoop} &= \frac{(8-e^2)((2-e)2a - 3t)}{(6+e)(2-e)(4+e-e^2)}, \\
 p_{x,noncoop} = p_{y,noncoop}^* &= \frac{(4+e)(2-e)(1-e)(2+e)a + 2e(7+e-e^2)t}{(6+e)(2-e)(4+e-e^2)}, \\
 p_{x,noncoop}^* = p_{y,noncoop} &= \frac{(4+e)(2-e)(1-e)(2+e)a + (24-13e^2+e^3(2+e))t}{(6+e)(2-e)(4+e-e^2)}, \\
 w_{noncoop} = w_{noncoop}^* &= b + \frac{(64-24e^2+2e^4)2a - (48+4e-e^2(3-e)(4+e))t}{4(6+e)(4+e-e^2)},
 \end{aligned}$$

$$\begin{aligned}
x_{coop1} = y_{coop1}^* &= \frac{(2-e)2a + (1-2e)t}{2(2-e)(1+e)}, & x_{coop1}^* = y_{coop1} &= \frac{(2-e)2a - 3t}{2(2-e)(1+e)}, \\
p_{x,coop} = p_{y,coop}^* &= \frac{t}{2(2-e)}, & p_{x,coop}^* = p_{y,coop} &= \frac{(3-2e)t}{2(2-e)}, \\
w_{coop1} = w_{coop1}^* &= b + \frac{(2a-t)(4-e^2)}{4(1+e)}.
\end{aligned}$$

### A.3 Proof of Proposition 2

Notice first that for our assumption of positive levels of intra-industry trade to hold in equilibrium, trading costs  $t$  have to be lower than  $\frac{(2-e)2a}{3}$  (see Appendix A.2). It then follows directly from (44) that  $s_{coop1} - s_{noncoop}$  is positive for  $0 \leq e < 1$  and zero for  $e = 0$ .

Moreover, differentiating  $s_{coop1} - s_{noncoop}$  with respect to  $e$  yields:

$$\frac{\partial(s_{coop1} - s_{noncoop})}{\partial e} = \frac{-(4-e)(2+e)(2a(2-e)\kappa_1 - 3t\kappa_2)}{4(2-e)^2(1+e)^2(6+e)^2(4+e-e^2)^2},$$

where  $\kappa_1 = (2-e)(368 + 252e - 66e^2 - 64e^3 + e^4(14e + e^2 - 1))$  and  $\kappa_2 = 544 + 8e - 148e^2 + e^3(94 + 15e - 9e^3)$ . It then follows from  $\kappa_1 \geq \kappa_2$  for  $e \in [0, 1]$  and  $(2-e)2a > 3t$  that  $s_{coop1} - s_{noncoop}$  is strictly decreasing in  $e$ .

Finally, differentiating the welfare gain from cooperation with respect to  $e$  yields:

$$\begin{aligned}
&\frac{\partial(W_{coop1} + W_{coop1}^* - W_{noncoop} - W_{noncoop}^*)}{\partial e} = \\
&\frac{-(4-e)(1-e)(2+e)(2a(2-e) - 3t)(2a(2-e)\lambda_1 - 3t\lambda_2)}{2(2-e)^3(1+e)^2(6+e)^3(4+e-e^2)^3}
\end{aligned}$$

with  $\lambda_1 = (2-e)(640 + 512e - 116e^2 - 72e^3 + 31e^4 + 14e^5 - e^6)$  and  $\lambda_2 = 896 + 128e - 272e^2 + 284e^3 + 40e^4 - 61e^5 - 10e^6 + 3e^7$ . It then follows from  $\lambda_1 \geq \lambda_2$  for  $e \in [0, 1]$  and  $(2-e)2a > 3t$  that the welfare gain from cooperation is decreasing in  $e$ .

## B Appendix to Section 5.2

### B.1 Proof of Proposition 3

Notice first that in the non-cooperative equilibrium exports from Home to Foreign and vice versa are given by  $x = y = \frac{4a-3t}{6}$ . For our assumption of positive intra-industry trade to hold in equilibrium, trading costs  $t$  thus have to be lower than  $4/3a$ . Recall further that  $b - b^*$  is positive by assumption. We can now consider

each part of Proposition 3 in turn:

- i. Calculating the aggregate welfare gain from policy harmonisation  $\Delta(W + W^*) = W_{coop1} + W_{coop1}^* - (W_{noncoop} + W_{noncoop}^*)$  yields:

$$\Delta(W + W^*) = \frac{(8a - 6t)^2 - 9(b - b^*)^2}{288}.$$

Next, observe that

$$\lim_{b-b^* \rightarrow 0} \Delta(W + W^*) = \frac{(8a - 6t)^2}{288} > 0, \quad \lim_{b-b^* \rightarrow \infty} \Delta(W + W^*) = -\infty,$$

and

$$\frac{\partial \Delta(W + W^*)}{\partial (b - b^*)} = \frac{-(b - b^*)}{16} < 0.$$

It then follows that there exist a threshold  $b'$  such that the aggregate welfare gain from policy harmonisation is positive (negative) iff  $b - b^* < (>) b'$ . It is easily verified that the threshold equals  $\frac{2(4a-3t)}{3}$ .

- ii. Calculating Home's welfare gain from policy harmonisation  $\Delta W = W_{coop1} - W_{noncoop}$  yields:

$$\Delta W = \frac{(8a - 6t)^2 - 9(b - b^*)^2 + 36(4a - 3t)(b - b^*)}{576}.$$

Next, observe that

$$\lim_{b-b^* \rightarrow 0} \Delta W = \frac{(8a - 6t)^2}{576} > 0, \quad \lim_{b-b^* \rightarrow \infty} \Delta W = -\infty,$$

and

$$\begin{aligned} \frac{\partial \Delta W}{\partial (b - b^*)} &= \frac{-(b - b^*) + 2(4a - 3t)}{32}, \\ \frac{\partial^2 \Delta W}{\partial^2 (b - b^*)} &= -\frac{1}{32}. \end{aligned}$$

Home's gain from policy harmonisation is thus strictly positive as  $b - b^*$  approaches zero but negative for large cross-country differences in the unemployment benefit system. We furthermore see that  $W_{coop1} - W_{noncoop}$  first increases in  $b - b^*$ , reaches a maximum at  $b - b^* = 8a - 6t$  and decreases thereafter. It then follows that there exist a threshold  $b''$  such that the welfare gain from policy harmonisation in Home is positive (negative) iff  $b - b^* < (>) b''$ . It is

easily verified that the threshold equals  $(\sqrt{10} + 3)\frac{2(4a-3t)}{3}$ .

- iii. Calculating Foreign's welfare gain from policy harmonisation  $\Delta W^* = W_{coop1}^* - W_{noncoop}^*$  yields:

$$\Delta W^* = \frac{(8a - 6t)^2 - 9(b - b^*)^2 - 36(4a - 3t)(b - b^*)}{576}.$$

Next, observe that

$$\lim_{b-b^* \rightarrow 0} \Delta W^* = \frac{(8a - 6t)^2}{576} > 0, \quad \lim_{b-b^* \rightarrow \infty} \Delta W^* = -\infty,$$

and

$$\frac{\partial \Delta W^*}{\partial (b - b^*)} = \frac{-(b - b^*) - 2(4a - 3t)}{32} < 0.$$

It then follows that there exist a threshold  $b'''$  such that the welfare gain from policy harmonisation in Foreign is positive (negative) iff  $b - b^* < (>) b'''$ . It is easily verified that the threshold equals  $(\sqrt{10} - 3)\frac{2(4a-3t)}{3}$ .

- iv. Subtracting  $W_{coop1}^* - Z$  from  $W_{coop1} - Z^*$  gives

$$\frac{(b - b^*)(4a - 3t)}{8},$$

which is always positive for  $b > b^*$ .

## C Appendix to Section 6.1

### C.1 Proof of Proposition 4

Notice first that in the (optimal) cooperative equilibrium exports from Home to Foreign and vice versa are given by  $x = y = \frac{4a-3t}{4}$ . For our assumption of positive intra-industry trade to hold in equilibrium, trading costs  $t$  thus have to be lower than  $4/3a$ . Recall further that  $b - b^*$  is positive by assumption. We now consider each part of Proposition 4 in turn:

- i. Calculating  $W_{coop2} - W_{coop1}$  yields:

$$W_{coop2} - W_{coop1} = \frac{(b - b^*)(b - b^* - (16a - 12t))}{64}.$$

Next, observe that

$$\lim_{b-b^* \rightarrow 0} W_{coop2} - W_{coop1} = 0, \quad \lim_{b-b^* \rightarrow \infty} W_{coop2} - W_{coop1} = \infty,$$

and

$$\begin{aligned} \frac{\partial(W_{coop2} - W_{coop1})}{\partial(b - b^*)} &= \frac{b - b^* - (8a - 6t)}{32}, \\ \frac{\partial(W_{coop2} - W_{coop1})}{\partial^2(b - b^*)} &= \frac{1}{32}. \end{aligned}$$

Home's gain from optimal policy cooperation (relative to policy harmonisation) is thus strictly positive for large cross-country differences and converges to zero as  $b - b^*$  approaches zero. We furthermore see that for  $b - b^* < 8a - 6t$   $W_{coop2} - W_{coop1}$  decreases in  $b - b^*$ , then reaches a minimum at  $b - b^* = 8a - 6t$  and increases thereafter. It then follows that there exist a threshold  $b'$  such that  $W_{coop2} - W_{coop1} < (>)0$  iff  $b - b^* < (>) b'$ . It is easily verified that the threshold  $b'$  equals  $4(4a - 3t)$ .

ii. Calculating  $W_{coop2}^* - W_{coop1}^*$  yields:

$$W_{coop2} - W_{coop1} = \frac{(b - b^*)(b - b^* + (16a - 12t))}{64},$$

which is always positive for  $b - b^* > 0$ .

## D Appendix to Section 6.2

### D.1 Reaction Functions and Second-Order Conditions

If governments choose the level of wage subsidies non-cooperatively, the reaction function  $s(s^*)$  for Home will read:

$$s = \frac{4es^* + 4b(8 - e - e^2) - (2 - e)(4 + e)(2a - t)}{4(8 - e^2)}. \quad (69)$$

The system of reaction functions is stable as we have  $\left| \frac{\partial s(s^*)}{\partial s^*} \right| = \frac{e}{8 - e^2} < 1$  for  $e \in [0, 1]$ . Moreover, the second order-conditions for a maximum are also fulfilled:

$$\frac{\partial^2 VE}{\partial^2 s} = \frac{\partial^2 VE^*}{\partial^2 s^*} = \frac{-8(8 - e^2)}{64 - 20e^2 + e^4} < 0. \quad (70)$$



If governments agree to cooperate, the second-order condition will be given by

$$\frac{\partial^2(V E + V E^*)}{\partial^2 s} = \frac{-16}{(4 - e)(2 + e)} < 0. \quad (71)$$