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Revisiting the Comovement Puzzle: the Input-Output Structure as an Additional Solution

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Revisiting the comovement puzzle: the Input-Output structure as an additional solution*

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Abstract

We propose an additional solution to the comovement puzzle by developing a two-sector monetary model with housing production and an input-output structure. The model generates comovement between consumption and residential investment for large range of shocks hitting the economy. Consistent with previous work, we find that our model produces highly persistence responses in aggregate consumption, aggregate output and residential investment. We show that the results are highly robust to different policy rule specifications. We find that the lower the labour shares, the higher the relative volatility of residential investment. The model with an IO structure is works under different specifications of the period utility function. We extend the model to allow for wage rigidities and show that our proposed solution can perfectly work alongside previous ones.

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Contents

1 Introduction

Previous research shows that monetary shocks in an two-sector economy with durable production and sticky prices generate a "comovement puzzle"[1] in sectoral outputs. Several solutions have been found for this problem. Focusing on the supply side, we provide an additional solution to the puzzle by constructing a two-sector New Keynesian (NK) model with numeraire production, housing production and an Input-Output structure (IO). We argue that comovement can be obtained by means of assuming that the material inputs are subject to same price rigidity as final consumption. Our results show that the real marginal cost in the housing sector inherits the price rigidity of the numeraire sector, which implies that producing new houses becomes more costly after a monetary policy contraction. Although we restrict our analysis to housing, but we argue that the results are equally applicable to an environment with durable production. We assume that, if material inputs are subject to nominal price rigidities, the real marginal cost and, consequently, house prices also display such characteristics. The model delivers high persistence in the responses to aggregate consumption, residential investment and aggregate output. In an extension to our analysis, we introduce nominal wage rigidities into the IO framework with purpose of showing that alternative solutions to the puzzle can potentially complement one another.

Using the VAR approach, Erceg and Levin (2006) report a large response of durable consumption to an exogenous change in the nominal interest rate. In particular, they show that a monetary contraction generates a decline in residential investment that is ten times as sharp as the decline in aggregate ouput. The high sensitivity of residential investment is due to the fact that the stock of housing is large in comparison with the annual production of new housing[2]. In the same study, the authors develop a two-sector New Keynesian model that features equal degree of price rigidity across sectors, using the empirical findings to calibrate the structural parameters of the model. However, the assumption on price stickiness in their model is inconsistent with the findings obtained by Bils and Klenow (2004), where it is shown that the degree of stickiness in the durable sector is lower than that in the non-durable sector.

In reconciling such stylised facts, Barsky, House, and Kimball (2007) (BHK) build a general equilibrium model to show that the responses of sectoral outputs to a monetary shock depend on the assumption made on the cross-sectoral degree of rigidity of nominal prices[3]. If durable prices are flexible and non-durable prices sticky, then a monetary contraction leads to a decline in the non-durable output and to a rise in durable output.

[1]The comovement puzzle is a term that has been coined by the work of Barsky, House, and Kimball (2007).

[2]The study shows that monetary innovations have little impact on the flow to stock ratio and that the marginal utility of durables is nearly constant. From the Euler equation of housing, a fall in the prices of the durable goods is offset by an increase in the marginal utility of non-durable consumption.

[3]Their idea builds on the work by Ohanian, Stockman, and Kilian (1995), who develop a monetary model in which the degree of price stickiness differs across sectors.

This lack of comovement is clearly at odds with the empirical findings described above. The durable sector responds indirectly because the contraction in the non-durable sector gives rise to a reduction in the real marginal cost of the durable sector and to an expansion in the production of durable goods. At the source of the comovement puzzle lies the sharp reaction of real wages to the monetary innovation, which ultimately affects the marginal cost in the housing sector. Lower production costs in the durable sector result in a higher production of the durable good. According to their work, the comovement puzzle arises because the monetary innovation affects the sector with prices rigidities. The study shows that this theoretical results are clearly at odds with the empirical evidence. In an environment where the frequency of price adjustment is higher that in the non-durable sector, their study suggests two possible solutions exist to solve such puzzle and these are: nominal wage rigidities and credit constraints.

Carlstrom and Fuerst (2006) (CF) postulate that the introduction of nominal wage rigidities into the BHK framework can solve the comovement problem. Nominal rigidities *á* la Erceg, Henderson, and Levin (2000) reduce the flexibility of the real marginal cost in the housing sector. Their study proposes a further solution for the puzzle, which is to impose borrowing constraints - a hold up constraint - on the amount of new housing that can be purchased out of some fixed proportion of current labour income. This second alternative solution is less successful at solving the puzzle than the first one because it generates highly volatile wages. The authors introduce adjustment costs in the durable sector to generate reasonable volatilities in durable output. An alternative solution to the comovement is proposed in a recent study by Kitamura and Takamura (2008) where prices are subject to sticky information. Ultimately, the supply side models - included our own - rely on the idea that the real marginal cost in the housing sector inherits some degree of nominal stickiness.

There is another strand of work that focuses on a demand side to solve the comovement puzzle. Monacelli (2008), and Iacoviello and Neri (2007) argue that sectoral comovement in the housing market can be explained by using housing to secure borrowing. This line of research relies on the idea that credit markets are subject to informational asymmetries between borrowers and lenders. This type of modelling device delivers comovement between aggregate consumption and residential investment to monetary innovations. To motivate equilibrium borrowing and lending, such suite of models assume that borrowers discount their future at much lower rate than lenders. This heterogeneity in thrift means that borrowers would always like to consume more today, using housing as collateral to secure debt.

Our analysis is presented in the following order. Section 2 introduces the main structure of the model, it sets up the economic problems of the agents that operate in the economy to derive the optimal choices for each of the agents operating in the economy. Section 3 compares the results of the model with the findings obtained in previous work. We present our results in such way so as to have a reference point from where to start our analysis. We carry out a robustness check for different Taylor Rule specifications and we do a sensitivity analysis on the parameter that, we believe, governs the cross-sectoral output comovement. In section 4, we improve on the comovement puzzle in inputs by extending the model to account for nominal wage rigidities. Section 5 concludes.

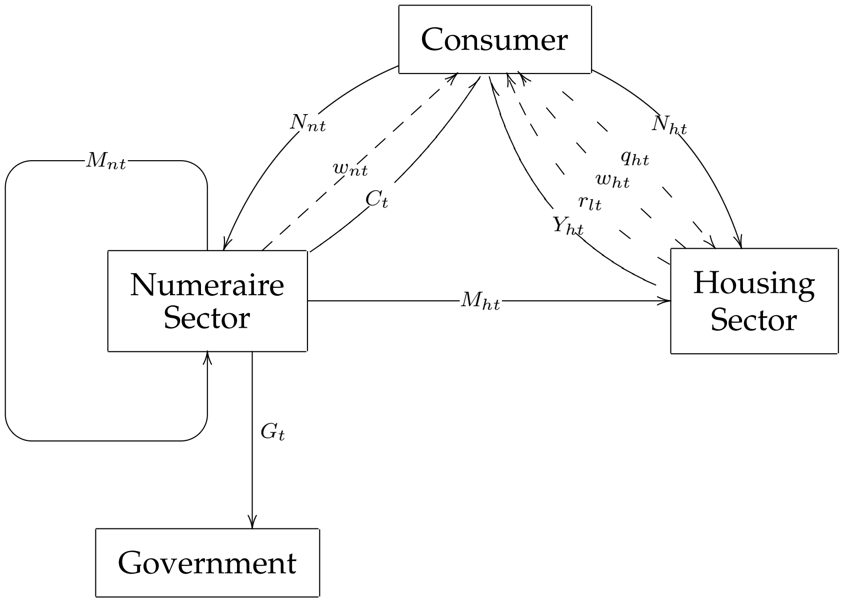
2 The Model

The model is a closed-economy NK model, along the lines of Woodford (2003), driven by a series of productivity, monetary and government innovations. The difference with the standard literature is that we develop a two-sector model with numeraire production and housing production. The driving force of our model is the introduction of an IO structure, which loosely resembles the work by Davis and Heathcote (2005). The production structure in our model is more closely related to the work by Bouakez, Emanuela, and Ruge-Murcia (2005) and Moro (2007). As stated in the diagram below, the main difference with such studies however is the introduction of housing production and consumption. Our model features a representative consumer that derives utility from housing services but, at the same, he or she receives a return for investing in housing. In terms of the productive structure, the difference with these studies is that, while the numeraire sector produces final goods and material inputs, the housing sector produces new houses. We work under the simplifying assumption that the aggregate composition of material inputs across sectors is the same[4] and that the housing sector does not produce material goods. A third difference is the introduction of internal habits to match hump-shaped responses of consumption to monetary and real innovations. We use this modelling device because it reduces considerably the volatility of residential investment and it allows us to compare our results with previous studies. Moreover, our model features a passive government that purchases numeraire goods via the collection lump-sum taxes and debt issuance. The model abstracts from capital, we conjecture that the introduction of capital would not substantially change our results.

We assume complete markets, which allows us to work in an representative agent environment. The model features the following economic agents: a representative consumer, a continuum of numeraire firms, a representative housing producing firm and government/ monetary authority. To produce numeraire output, Y_{nt}, and new houses, Y_{ht}, firms combine intermediate inputs, M_{ht} and M_{nt}, with labour, N_{nt} and N_{ht} respectively. Firms demand sector-specific labour and pay nominal wages W_{nt} and W_{ht}. We assume monopolistic competition in the numeraire sector to motivate price stickiness in consumer and material input prices. Firms in both sectors take the prices of the material inputs as given at the aggregate price P_t. On aggregate, firms in the numeraire sector produce final goods, C_t and G_t, which are sold to the representative consumer and the government at the exogenously given price P_t. New houses, Y_{ht}, are produced by the housing sector, which are sold to the representative consumer at the real price q_{ht}. Throughout our study, the variable in small letters q_{ht}, w_{nt} and w_{ht} will denote the prices of new houses and the prices of sectoral wages in terms of the numeraire good. The following diagram

[4]To assume otherwise will require making extensive changes to our baseline model.

illustrates the flow of goods and services and the payments in real terms in the model economy. The representative consumer owns a fixed amount of land, L_t, which is rented to the housing sector at the rental price r_{lt}.



The diagram is not exhaustive of all the flows and payments that take place in our economy. In particular, this diagram abstracts from dividend payments, taxation and government debt. The representative consumer owns the firms in the numeraire sector, so he or she receives an average dividend for the amount Φ_t. The consumer pays taxes T_t and buys short-term debt B_t, which gives a nominal gross return R_t that is set by the monetary authority (a government agency) according to an inertial Taylor rule.

2.1 The Consumer

The preferences of the representative consumer are defined over the consumption aggregator, $X_t = (C_t - bC_{t-1})^{\zeta} (\kappa H_t)^{1-\zeta}$ and the supply of per capita labour services, $N_t = \left(N_{nt}^{1+\xi} + N_{ht}^{1+\xi}\right)^{\frac{1}{1+\xi}}$. The consumption aggregator is a standard Cobb-Douglas combining aggregate consumption, C_t, and housing services, κH_t[5]. Aggregate consumption is subject to internal habits, where b represents the degree of persistence. We assume that housing services are proportional to the housing stock, H_t, by the amount κ and that ζ denotes the share of numeraire consumption in the consumption aggregator. In the model,

[5] It is important that the reader distinguishes between the consumption aggregator, X_t, and aggregate consumption C_t

labour mobility is restricted across sectors, where we use the parameter ξ to control the degree of complementarity in hours.

Aggregate consumption is assumed to be a composite of a continuum of differentiated goods $C_t(i)$ indexed by $i \in [0,1]$ via

$$C_t \geq \left[\int_0^1 C_t^{1-1/\epsilon}(i)\, di \right]^{1/(1-1/\epsilon)}$$

where ϵ denotes the intra-temporal elasticity of substitution across the different consumption varieties. In each period t, the consumer minimises his or her consumption expenditure $\int_0^1 P_t(i)\, C_t(i)\, di$ subject to the constraint above. The demand of the i consumption variety, $C_t(i)$, is.

$$C_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\epsilon} C_t$$

Here P_t denotes the aggregate price index of the aggregate consumption good, which is equal to

$$P_t = \left[\int_0^1 P_t^{1-\epsilon}(i)\, di \right]^{\frac{1}{1-\epsilon}}$$

The period utility of the representative consumer is

$$U(X_t, N_t) = \left(\frac{X_\tau^{1-\sigma} - 1}{1-\sigma} - \chi \frac{N_\tau^{1+\varrho}}{1+\varrho} \right)$$

where χ is the parameter that governs the cost of labour supply in utility terms. Although it is costly for the consumer to supply labour, he or she receives a monetary compensation in the form of wages from each sector in the economy. We make the assumption that, to consume housing services, the consumer requires purchasing numeraire goods. It has been shown in previous studies that the assumption of complementarity between housing services and consumption cannot solve the comovement puzzle on its own but it can help alleviating the problem. The results that of our study are independent of the degree of complementarity between housing and numeraire consumption. The budget constraint of the representative consumer expressed in real terms of the numeraire good is given by

$$\begin{aligned} C_t \quad + \quad & q_{ht}\left[H_{ht} - (1-\delta_h) H_{ht-1}\right] + q_{lt}(L_t - L_{t-1}) + B_t \leq \\ & \leq \frac{R_{t-1}}{\pi_t} B_{t-1} + w_{nt}N_{nt} + w_{ht}N_{ht} + r_{lt}L_{t-1} + \Phi_t + T_t \\ & \quad for = \{t, t+1, t+2 ...\} \end{aligned} \tag{1}$$

The term $H_{ht} - (1-\delta_h) H_{ht-1}$ denotes housing investment - i.e. residential investment - and δ_h the depreciation of the housing stock. In addition, the representative consumer owns a fixed stock of land, L_t, and receives a rental price, r_{lt}, from the housing sector. The maximisation problem of the representative consumer is given by

$$V_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left(\frac{X_\tau^{1-\sigma} - 1}{1-\sigma} - \chi \frac{N_\tau^{1+\varrho}}{1+\varrho} \right)$$

subject to (1) and a set of transversality and non-negativity conditions. The first order conditions resulting from the maximisation problem of the representative consumer are

$$\frac{\zeta X_t^{1-\sigma}}{C_t - bC_{t-1}} - \beta b \zeta \frac{X_{t+1}^{1-\sigma}}{C_{t+1} - bC_t} = \Lambda_t \tag{2}$$

$$\Lambda_t = \beta R_t E_t \frac{\Lambda_{t+1}}{\pi_{t+1}} \tag{3}$$

$$\Lambda_t q_{ht} = \frac{(1-\zeta) X_t^{1-\sigma}}{H_t} + (1-\delta_h) \beta \Lambda_{t+1} q_{ht+1} \tag{4}$$

$$\Lambda_t w_{nt} = \chi \left(N_{n\tau}^{1+\xi} + N_{h\tau}^{1+\xi} \right)^{\frac{1+\varrho}{1+\xi}-1} N_{nt}^{\xi} \tag{5}$$

$$\Lambda_t w_{ht} = \chi \left(N_{n\tau}^{1+\xi} + N_{h\tau}^{1+\xi} \right)^{\frac{1+\varrho}{1+\xi}-1} N_{ht}^{\xi} \tag{6}$$

$$\Lambda_t q_{lt} = \beta \Lambda_{t+1} r_{lt+1} + \beta \Lambda_{t+1} q_{lt+1} \tag{7}$$

where Λ_t is the lagrange multiplier and $\pi_t = P_t/P_{t-1}$ the gross inflation rate. Equation (2) sets to the marginal utility of consumption to the shadow value of the budget constraint, equation (3) is the Euler equation for aggregate consumption, equation (4) the Euler equation for housing, equations (5) and (6) state that the consumer is indifferent between working an extra unit of time and receiving the hourly sectoral wage to spend in an additional unit of consumption. Finally, equation (7) is the Euler equation for land holdings[6]. The non-arbitrage condition between government bonds and housing holds.

2.2 The Numeraire Sector

In our model economy, the firms operating in the numeraire sector produce material inputs and final goods, sell the former to various firms in the economy and the latter to the representative consumer and the government. Each firm i in the numeraire sector hires labour, $N_{nt}(i)$, at the real price w_{nt}, while buying material inputs from other firms in the sector, $M_{nt}(i)$ at price P_t to produce, $Y_{nt}(i)$. The variable $M_{nt}(i)$ is defined as a material

[6] As shown in Appendix B, this last condition is no longer valid when the re-specify the model to account for consumption durables rather than housing.

input aggregator that combines the varieties of the material inputs produced within the sector

$$M_{nt}(i) = \left[\int_0^1 \left[M_{nt}(i,j) \right]^{1-1/\epsilon} dj \right]^{1/(1-1/\epsilon)}$$

where $M_{nt}(i,j)$ is the within-sector demand of type j by firm i[7]. To find the optimal choice of firm i, we divide the minimisation problem into two separate intra-period problems. The first problem that firm i faces is to minimise the total cost associated to the purchase of material inputs, $\int_0^1 P_t(j) M_{nt}(i,j)$, subject to the material input aggregator given above. The demand for each specific variety of material input is

$$M_{nt}(i,j) = \left[\frac{P_t(j)}{P_t} \right]^{-\epsilon} M_{nt}(i)$$

where $P_t = \left[\int_0^1 \left[P_t(j) \right]^{1-\epsilon} dj \right]^{1/(1-\epsilon)}$ is the aggregate price of the material input paid by the numeraire sector. Aggregating over all firms in the sector yields

$$M_{nt}(j) = \left[\frac{P_t(j)}{P_t} \right]^{-\epsilon} M_{nt}$$

where $M_{nt} = \int_0^1 M_{nt}(i)\, di$. The second intra-period problem faced by firm i is to choose the aggregate demand for material inputs, $M_{nt}(i)$, and labour, $N_{nt}(i)$ to minimise total costs. Firm i faces the following problem

$$\min_{N_{nt}(i), M_{nt}(i)} \quad w_{nt} N_{nt}(i) + M_{nt}(i)$$

subject to

$$Y_{nt}(i) \leq Z_{nt} N_{nt}^{\alpha}(i) M_{nt}^{1-\alpha}(i) \tag{8}$$

where Z_{nt} follows

$$Z_{nt} = \rho_h Z_{nt-1} + \varepsilon_{nt}, \quad \varepsilon_{nt} \sim \mathbf{N}\left(0, \sigma_n^2\right) \tag{9}$$

The optimality conditions of firm i with respect to $N_{nt}(i)$ and $M_{nt}(i)$ are given by

$$\alpha \frac{Y_{nt}(i)}{N_{nt}(i)} = mc_t(i)\, w_{nt} \tag{10}$$

[7] Alternatively, we could assume that the material input aggregator is also a function of the material inputs produced by the housing sector. In this case, the material input aggregator takes the following form

$$M_{nt}(i) = \left[\omega_n \int_0^1 \left[M_{nt}^n(i,j) \right]^{1-1/\epsilon} dj + (1-\omega_n) \left[M_{nt}^h(i) \right]^{1-1/\epsilon} \right]^{1/(1-1/\epsilon)}$$

where $M_{nt}^n(i,j)$ is the within-sector demand of type j by firm i and $M_{nt}^h(i)$ is the demand of material inputs produced in the housing sector. We postulate however that it is not realistic to assume that the housing sector produces material goods that are used by the numeraire sector.

and

$$(1-\alpha)\frac{Y_{nt}(i)}{M_{nt}(i)} = mc_t(i) \tag{11}$$

where $mc_t(i)$ is the real marginal cost of firm i. The market clearing condition for labour is $N_{nt} = \int_0^1 N_{nt}(i)\,di$. In the symmetric equilibrium, all firms in the numeraire sector face the same marginal cost. By combining (8), (10) and (11), we derive the real marginal cost for the numeraire sector

$$mc_t = \frac{1}{A_{nt}} w_{nt}^{\alpha} \frac{1}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}$$

This expression states that the real marginal cost in the sector is positively related to the material input prices.

Following Calvo (1983) and Yun (1996), a fixed proportion of randomly chosen firms $\theta \in [0,1)$ is not allowed to reset their prices. The remaining proportion of firms, $1-\theta$, chooses the optimal price, P_t^*, that maximises the expected discounted value of nominal profits

$$\max_{P_t(i)} \quad \Phi_t(i) = E_t \sum_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} P_\tau (\beta\theta)^{\tau} \left[\frac{P_t(i)}{P_\tau} - mc_\tau\right] Y_{n\tau}(i)$$

Total aggregate profits are rebated to the consumer at time t by the amount $\Phi_t = \int_0^1 \Phi_t(i)\,di$. The first order condition with respect to $P_t(i)$ is given by

$$E_t \sum_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} P_\tau Y_{n\tau} \left(\frac{P_t^*}{P_\tau}\right)^{-\epsilon} (\theta\beta)^s \left[\left(\frac{\epsilon-1}{\epsilon}\right) - mc_\tau \left(\frac{P_t^*}{P_t}\right)\right] = 0 \tag{12}$$

or, equivalently, by

$$E_t \sum_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} P_\tau Y_{n\tau} \tilde{P}^{-\epsilon} (\theta\beta)^s \left[\left(\frac{\epsilon-1}{\epsilon}\right) - mc_\tau \tilde{P}_t\right] = 0 \tag{13}$$

where $\tilde{P}_t = P_t^*/P_t$. This expression states that firm i chooses the optimal price that sets the average future expected marginal revenues equal to the future average expected marginal costs[8]. Since a proportion of firms in the sector cannot set its price optimally at time t, the aggregate price level responds to the following law

$$1 = \left[\theta\left(\frac{1}{\Pi_t}\right)^{1-\epsilon} + (1-\theta)\left(\tilde{P}_t\right)^{1-\epsilon}\right]^{1/(1-\epsilon)} \tag{14}$$

[8]For the purpose of solving the model, we re-express (13) in a recursive forms following Schmitt-Grohe and Uribe (2004b).

2.3 The Housing Sector

We assume perfect competition and flexible prices in the housing sector to work in a representative agent environment. Given the input prices, the representative firm produces new houses at the real price q_{ht} using sectoral labour, N_{ht}, and material inputs, M_{ht}. Similarly, the variable M_{ht} is an aggregator of material inputs produced by of all the firms in the numeraire sector

$$M_{ht} = \left[\int_0^1 \left[M_{ht}(i) \right]^{1-1/\epsilon} di \right]^{1/(1-1/\epsilon)}$$

where the elasticity of substitution between the variety of material inputs is equal to the elasticity of substitution between consumption and material input varieties in the numeraire sector[9]. We omit here from stating the material input demands in the housing sector as such demands have a similar form to the material input demands in the numeraire sector. The representative housing producing firm takes input prices, w_{ht}, as given to solve the following maximisation problem

$$\max_{N_{ht}, M_{ht}} \quad p_{ht} Y_{ht}^{s} - w_{ht} N_{ht} - M_{ht} - r_{lt} L_{t-1}$$

subject to

$$Y_{ht} = Z_{ht} N_{ht}^{\eta} M_{ht}^{\gamma} L_{t-1}^{1-\eta-\gamma} \tag{15}$$

with Z_{ht}

$$Z_{ht} = \rho_h Z_{ht-1} + \varepsilon_{ht}, \quad \varepsilon_{ht} \sim \mathbf{N}\left(0, \sigma_h^2\right) \tag{16}$$

The first order conditions with respect to N_{ht}, M_{ht} and Y_{ht} are

$$\eta q_{ht} \frac{Y_{ht}}{N_{ht}} = w_{ht} \tag{17}$$

$$\gamma q_{ht} \frac{Y_{ht}}{M_{ht}} = 1 \tag{18}$$

[9] For the case of the housing sector, it would be more realistic to assume that the housing sector has a positive demand for the material inputs produced within the sector. An alternative material input aggregator in the housing sector could take the following form

$$M_{ht} = \left[\omega_h \int_0^1 \left[M_{ht}^{n}(i) \right]^{1-1/\epsilon} di + (1-\omega_h) \left(M_{ht}^{h} \right)^{1-1/\epsilon} \right]^{1/(1-1/\epsilon)}$$

In our model economy, we assume that inputs are only produced by the numeraire sector to simplify aggregation. The results of our model would change if we use this other aggregator because the real marginal cost in the housing sector would become be more flexible than in the specification of the model that we have chosen. However, the results may not be entirely different if we remove the housing production adjustment costs and

$$\left(1-\eta-\gamma\right)\frac{Y_{ht}}{L_{t-1}}=r_{lt} \tag{19}$$

Under perfect competition, the price of housing in real terms is equal to real marginal cost. By combining equations (15), (17), (18) , we derive an expression for the equilibrium marginal cost in the housing sector

$$q_{ht}=mc_{ht}=\left[\frac{1}{Z_{ht}}\left(\frac{w_{ht}}{\eta}\right)^{\eta}\left(\frac{1}{\gamma}\right)^{\gamma}\right]^{\frac{1}{\eta+\gamma}}$$

As it will be shown later, sectoral comovement stems from the behaviour of the real marginal cost in the housing sector. The real marginal cost is a function of wages, which are assumed to be flexible. In the standard two-sector NK model with flexible wages, the marginal cost is a linear function of real wages. In our model however, the real marginal cost is non-linear in the real wages.

2.4 The Government

The government finances its expenditure through lump-sum taxation and by issuing government bonds. The government budget constraint is

$$G_t=B_t-\frac{R_{t-1}}{\Pi_t}B_{t-1}+T_t$$

and government expenditure

$$G_t=G^{1-\rho_g}G_{t-1}^{\rho_g}e^{\varepsilon_{gt}},\qquad \varepsilon_{gt}\sim\mathbf{N}\left(0,\sigma_g^2\right) \tag{20}$$

where G_t denotes the real government expenditure, G the steady state value of government purchases and ρ_g the persistence of the government shock. We assume that the government minimises the cost of producing G_t with the public demand for a particular variety of good i, $G_t\left(i\right)$, being

$$G_t\left(i\right)=\left[\frac{P_t\left(i\right)}{P_t}\right]^{-\epsilon}G_t$$

where aggregate government expenditure is defined as $G_t=\int_0^1 G_t\left(i\right)di$.

A government agency, also known as the monetary authority, sets the nominal interest rate following an interest rate rule of the form

$$R_t=R_{t-1}^{\mu_r}\Pi_t^{\mu_\Pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\mu_Y}\bar{R}^{1-\mu_r}\varepsilon_{rt} \tag{21}$$

where aggregate output is defined as $Y_t = C_t + G_t + Q_{ht}Y_{ht}$, and ε_{rt} is an iid shock[10]. Monetary policy responds systematically to the contemporaneous inflation and to deviations of output from past values. The policy rule accounts for some degree of interest rate smoothing as in CF. The fact that the Taylor Rule is inertial implies that i.i.d shocks to the nominal interest rate have long-lasting effects on the responses of the main aggregate.

2.5 The Market clearing Conditions and Equilibrium

We work under the assumption that all firms in the numeraire sector are symmetric, so the following condition holds

$$M_{nt}(j,i) = \left[\frac{P_t(i)}{P_t}\right]^{-\epsilon} M_{nt}(j)$$

where $M_{nt}(j,i)$ is the within-sector demand of type i by firm j. This condition is symmetric to the one given in previous sections. In equilibrium, the demand of each variety i must be equal to the supply of i $Y_{nt}(i) = C_t(i) + G_t(i) + \int_0^1 M_{nt}(j,i)\,dj + M_{ht}(i)$. By replacing the individual demands of each variety i into the equilibrium condition in the numeraire sector yields

$$Y_{nt}(i) = \left[C_t + G_t + \int_0^1 M_{nt}(j)\,dj + M_{ht}\right]\left[\frac{P_t(i)}{P_t}\right]^{-\epsilon}$$

Summing across of firms i gives

$$Y_{nt} = (C_t + G_t + M_{nt} + M_{ht}) \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\epsilon} di$$

with $M_{nt} = \int_0^1 M_n(j)\,dj$. Note that the second term on the right hand side of the above expression is the price dispersion in the economy. The price dispersion must be taken into account only if interested in carrying out optimal monetary policy or higher than first order approximations. As we solve the model using a first order approximation, we have that

$$Y_{nt} = C_t + G_t + M_{nt} + M_{ht} \tag{22}$$

This market clearing condition states that the aggregate production of numeraire goods is equal to the sum of aggregate consumption, aggregate government expenditure and the aggregate demands of material inputs from both sectors. The second market clearing condition is

$$H_{ht} - (1-\delta_h)\,H_{ht-1} = Y_{ht} \tag{23}$$

[10]The measure of personal consumption C_t excludes housing services.

In equilibrium, residential investment is equal to the production of new houses. We assume that land is in fixed supply and normalise the value of land to the value of residential investment in the steady state

$$L_t = \bar{Y}_h \tag{24}$$

A stationary competitive equilibrium is a set of endogenous stationary processes, Λ_t, C_t, H_t, N_{nt}, N_{ht}, L_t, R_t, Π_t, q_{ht}, r_{lt}, W_{nt}, W_{ht}, M_{nt}, M_{ht}, Y_{nt}, mc_{nt}, Y_{ht}, $\tilde{P}_t$ and exogeneous stochastic process $\{Z_{nt}, Z_{ht}, Z_{mt}, G_t\}_{t=0}^{\infty}$ satisfying (2)-(24) given the initial conditions for C_{-1}, H_{-1}, R_{-1}.

3 The Deterministic Steady State

For the computation of the deterministic steady state, we remove the time subscript from the equilibrium equations to find the values of the aggregates. The variables without time subscript denote in this section the value of the variable in the deterministic steady state. We work under the assumption that the gross inflation rate, Π, is equal to zero in the deterministic steady state. Under this assumption, it follows from equation (3) that, the real gross interest rate is $R = 1/\beta$. Government bonds are in zero net supply in the steady state.

To make the computation of the steady state simpler and analytically tractable, we normalise the fixed stock of land, L, to the steady state value of residential investment Y_h[11]. This normalisation simplifies the calculation of the steady state. In the steady state, combining equations (8), (10) and (11) yields the following expression for the real wage in the numeraire sector.

$$w_n = \alpha \left(1-\alpha\right)^{\frac{1-\alpha}{\alpha}} mc_n^{\frac{1}{\alpha}} \tag{25}$$

Government expenditure is $\bar{g} * Y$, where $\bar{g}$ is the ratio of government expenditure to output. Using this shorcut, we can re-express (22) as

$$Y_n = C + \bar{g} * (Y_n - M_n - M_h + q_h Y_h) + M_n + M_h + q_h Y_h \tag{26}$$

which combined with (2), (4), (11), (18) and (23) yields

$$Y_n = \frac{1 + \left[\left(1+\bar{g}\right)\gamma + \bar{g}\right]\delta A}{1 - \left(1-\bar{g}\right)\left(1-\alpha\right) mc_n - \bar{g}} C \tag{27}$$

where $A = \frac{(1-\zeta)(1-b)}{\zeta(1-b\beta)[1-\beta(1-\delta)]}$. Moreover, dividing (5) and (6) we obtain the following expression for the real wage in the housing sector

$$w_h = w_n \left(\frac{N_h}{N_n}\right)^{\xi} \tag{28}$$

[11] In Appendix B, we show an alternative steady state normalising the stock of land to 1. We find that adopting this alternative normalisation does not alter the results.

Substituting equations (4), (10), (17) into (28) yields

$$w_h = w_n \left(\frac{\eta \delta A C}{\alpha m c_n Y_h} \right)^{\frac{\xi}{1+\xi}} \tag{29}$$

where C cancels with its counterpart in the denominator. The value of w_h simply depends on the parameters of the model. Subsequently, we derive an expression for the aggregate wage that is independent of any aggregate as

$$w = \left(w_n^{\frac{1+\xi}{\xi}} + w_h^{\frac{1+\xi}{\xi}} \right)^{\frac{\xi}{1+\xi}} \tag{30}$$

By combining the labour aggregator with equations (5), (6), (10) and (17) yields the following expression from which we can derive the value of aggregate consumption C

$$wN = \left\{ \alpha m c_n \frac{1 + \left[(1+\bar{g}) \gamma + \bar{g} \right] \delta A}{1 - (1-\bar{g})(1-\alpha) m c_n - \bar{g}} + \eta A \delta \right\} C \tag{31}$$

where N is normalised to 1. Rearranging,

$$C = \frac{wN}{\left\{ \alpha m c_n \frac{1+[(1+\bar{g})\gamma + \bar{g}]\delta A}{1-(1-\bar{g})(1-\alpha) m c_n - \bar{g}} + \eta A \delta \right\}} \tag{32}$$

By combining equations (15),(17), (18) and (19) we obtain the real value of house prices in the steady state

$$q_h = \left(\frac{w_h^{\eta}}{\eta^{\eta} \gamma^{\gamma}} \right)^{\frac{1}{\eta+\gamma}} \tag{33}$$

From (4), we can recover the value of the housing stock in the steady state

$$H = \frac{AC}{q_h} \tag{34}$$

and using (23) the value of residential investment

$$Y_h = \delta H \tag{35}$$

Finally, the value of all the remaining aggregates can be easily recovered by simple substitution.

4 Model Simulation

In this section, we compare our results against two different model specifications. We test our model against the Comovement Puzzle (CP) model as developed by BHK and the model economy proposed by CF. To compare our results with theirs, we assume internal habits and a passive government in all three specifications. The inclusion of habits has the following effects: it dampens the volatility of residential investment and it produces a hump-shaped response in numeraire consumption. The inclusion of habits alters the dynamics of the real marginal cost of production one new house while imposing a future cost to consumption. Our analysis focuses on housing rather than on durables, so the choice of all preference and production parameters is in line with empirical evidence and other related studies. We also carry out a robustness check for the specification of the Taylor Rule and the production parameters.

To compare our results with previous studies, we maintain the assumption of flexible house prices and sticky numeraire prices. It is worth noting that the model specifications are not taken to be the exact replica of the specifications in BHK and CF. We identify five features that describe the three models presented here, and these are: labour mobility, nominal wage rigidities, IO structure and adjustment cost in the production of new housing. Table 1 summarises the features characterising each of the three specifications that we analyse here

Table 1: **Model Characteristics**

Model	**LM**	**NWR**	**IO**	**AC**	**Land**
CP	✓	X	X	X	X
CF	✓	✓	X	✓	X
IO	X	X	✓	X	✓

The first model specification - the CP model - presents us with the comovement problem[12] and the last two specifications - the CF and IO[13] models - provide alternative solutions to the puzzle. We solve the various model specifications by means of implementing the first order approximation proposed by Schmitt-Grohe and Uribe (2004a).

4.1 Calibration

To find a numerical solution to the model, we set the inter-temporal elasticity of substitution, σ, to 2. The share of consumption in the utility function is set to $\zeta = 0.85$ such that the ratio of housing investment to output is around 6%. The degree of internal habit persistence, b, is set to a lower value relative to the estimates provided in the macro literature. The reason for doing so is that choosing higher values of b alters considerably the marginal rate of substitution between aggregate consumption and the supply of numeraire labour. The main reason for introducing habits in our model is necessarily to

[12] A model with imperfect mobility in labour under such model specification, the comovement problem in sectoral outputs is dampened but not resolved

[13] In Appendix A we propose an alternative model specification that can account for durable consumption instead of housing production.

compare our results with previous studies. The parameter κ has no other role than to match the steady state level of housing to numeraire consumption, which we set equal to 0.05.

Following Iacoviello and Neri (2007), we calibrate the inverse of the Frisch labour supply elasticity ϱ to 0.5. The value of χ is chosen such that the labour supply aggregator N_t is equal to 1 in the steady state. We adopt the methodology implemented by Horvath (2000) to calibrate ξ. The author finds that a greater value of ξ in relation to ϱ implies higher degree of complementarity between hours between the sectors of production. We assume that the value of ξ is unity, so that hours react less to the wage dispersion across sectors. The real interest rate is set to an annual rate of 2%, which corresponds to a quarterly discount rate, β, of 0.9926. We follow Calza, Monacelli, and Stracca (2007) in setting the depreciation of housing is δ_h to 0.5% quarterly, which corresponds to an annual depreciation rate of 2%. A low depreciation of the housing stock gives a high housing stock to residential investment ratio.

Iacoviello and Neri (2007) estimate a two-sector model DSGE with housing production using bayesian techniques but in doing so they calibrate the share of material inputs in the housing production function to 0.01. A recent study by Bouakez, Emanuela, and Ruge-Murcia (2005) shows that the estimate of the input share is larger than the one used by Iacoviello and Neri (2007). We average out the shares estimated by Bouakez, Emanuela, and Ruge-Murcia (2005) to calibrate the share of labour in numeraire production and set the value of α to 0.35. Following Schmitt-Grohe and Uribe (2004a), we assume that the steady-state markup in the numeraire sector is in the order of 20%, which implies a price elasticity of demand of 6.

Using U.S. BLS data, Bils and Klenow (2004) estimate that the median firm changes prices every 4.3 months. Moro (2007) finds that, in a model with an IO structure, the material input prices inherit the stickiness of the numeraire prices, which signifies more persistent responses of aggregate consumption and aggregate output than in the standard NK model. His study argues that this result corresponds to the Calvo parameter in the order of 0.3, which matches the average waiting time as estimated by Bils and Klenow (2004). The share of labour in the housing sector is proxied by using the results obtained by Bouakez, Emanuela, and Ruge-Murcia (2005) for the construction sector. We make the realistic assumption that the construction sector is more labour intensive than the rest of the economy and choose a labour share, η, in the order of 0.42.

The steady state share of government purchases to aggregate output is in the order of 25% due the fact that we abstract from capital formation. We consider a simple Taylor rule with the following weights: $\mu_\Phi = 0.5$, $\mu_Y = 0$ and $\mu_R = 0.7$. The persistence of the technological and government shocks are $\rho_n = 0.95$, $\rho_h = 0.95$ and $\rho_g = 0.95$ respectively. We set steady state inflation to 1. Table 2 provides a summary of the values of the main parameters under the three different model specifications.

Table 2: **Structural Parameters**

Parameter	CP	CF	IO	Description
β	0.9926	✓	✓	Discount Factor
b	0.4	✓	✓	Habit Persistence
ζ	0.85	✓	✓	Numeraire Consumption Share
κ	0.05	✓	✓	Housing Consumption Parameter
ϱ	1	1	2	Frisch Elasticity
χ	0.05	✓	✓	Preference Parameter
ξ	0	0	1	Inverse of ES in Sectoral Labour
G/Y	0.25	✓	✓	Government to GDP Ratio
δ_h	0.005	✓	✓	Housing Depreciation
α	1	1	0.35	Labour's Share in the Numeraire Production
η	1	1	0.42	Labour's Share in the Housing Production
φ	0	1	1	Short-Term Price Elasticity of Housing Supply
ϵ	6	✓	✓	Price Elasticity of Numeraire Demand
$1-\theta$	0.7	✓	✓	Proportion of Firms setting prices optimally
π^*	1	✓	✓	SS Inflation rate
μ_R	0.7	✓	✓	Baseline Policy Rule Parameter
μ_π	0.5	✓	✓	Baseline Policy Rule Parameter
μ_Y	0	✓	✓	Baseline Policy Rule Parameter
ρ_g	0.95	✓	✓	Shock Persistence of Gov. Exp.
ρ_n	0.95	✓	✓	Shock Persistence of Numeraire
ρ_h	0.95	✓	✓	Shock Persistence of Housing Sector

4.2 Results

We show that the comovement problem between consumption and residential investment disappears when we introduce an IO structure into the standard two-sector NK model. We separate the monetary shock from real shocks affecting the economy because we are interested in the analysing policy and in comparing our results with other studies. In addition, we analyse the behaviour of the model to three different policy rules in search for robust results. We carry out a sensitivity analysis on the parameter that are most relevant to explain comovement in our model. Finally, we check whether the model delivers the correct comovement independently of the source of the shock.

4.2.1 Monetary Innovation

Under the CP specification, a monetary innovation leads to a sharp contraction in house prices and to a rise in the the marginal utility of consumption, Λ_t, which is moderated by the introduction of habits and also by the fact that the consumer also wants to smooth the consumption of housing services. The increase in the marginal utility of consumption leads to a fall in numeraire consumption. The reason behind this offsetting effect is due to the near constancy of the marginal utility of housing. A large stock to flow ratio implies that the marginal utility is nearly constant given that the flow/stock ratio is small. Figure 1 shows the response of aggregate consumption to a monetary shock.

Price stickiness in the numeraire sector produces a fall in numeraire output, which is accompanied by a downward shift in the numeraire labour demand. The monetary contraction induces an increase in the real interest rate, which gives the representative consumer the incentive to exert more labour effort as the returns of savings rises. As numeraire output contracts, the demand for labour in the numeraire sector reduces. The assumption of perfect mobility in hours allows labour to move freely from the numeraire sector to the housing producing sector. Due to the fact that the numeraire sector is much larger than the housing sector, the reduction in aggregate labour demand, coupled with an expansion in the aggregate labour supply, puts downward pressure on the real wage and the real marginal cost in the housing sector. Therefore, producing new houses becomes less costly and more houses are produced in equilibrium.

Analytically, the problem of comovement problem arises because a temporary monetary shock has little impact on $\Lambda_t q_{ht}$. By iterating forward equation (4), we obtain an expression for this variable

$$q_{ht}\Lambda_t = E_t\left[\sum_{\tau=t}^{\infty}\left[\beta\left(1-\delta_h\right)\right]^{\tau-t}\frac{\left(1-\zeta\right)X_\tau^{1-\sigma}}{H_\tau}\right]$$

The assumption of non-separability of preferences implies that changes to the current and expected level of consumption and housing have a significant effect on the marginal utility of consumption. Note that in the model with habits, Λ_t responds according to (2), where consuming today has a future cost due to the presence of habits. Thus, the introduction of habits generates a lower degree of responsiveness to the marginal utility of consumption but it also alters the marginal rate of substitution between hours and consumption.

In the CP and CF models, the real marginal cost in the numeraire sector and house prices are equal. CF show that the introduction of nominal rigidities has the potential of solving the comovement puzzle. If nominal wages are sticky, then the real marginal cost in the housing sector, and equally the price of new housing, will inherit such rigidity. The dynamics of the real marginal cost in the housing sector will depend on the assumption about price stickiness. One caveat of introducing nominal wage stickiness in a two-sector model with labour as the only input of production is the high sensitivity of housing production to monetary shocks. To dampen such excess sensitivity of residential investment, CF introduce adjustment cost - assumption that we maintain in the generalised version of our model where we look at durables[14]. Similar to internal habits, the introduction of such modelling device on its own helps dampening the comovement problem.

[14]See Appendix A

Our analysis however presents two main important differences: the first one is that we allow for a very simple IO production structure and the second one is that we allow for complementarity in labour supply. The combination of these two features generates comovement between consumption and residential investment. A one-off rise in the policy rate leads to a rise in the real interest rate due to the fall in numeraire inflation. The return to savings increases such that the consumer has greater incentive to postpone aggregate consumption. In other words, the Lagrange multiplier Λ_t increases relative to its future value, which implies that the consumer would like to reduce the level of aggregate consumption at time t. Other things being equal, the rise in the real interest rate induces the consumer to supply more aggregate labour. In the wake of a monetary shock, a lower demand for aggregate consumption reduces the demand for labour. Real wages fall due to a lower labour demand and a higher labour supply. As material inputs become more expensive than labour, firms reduce their demand for material inputs. The extent to which the marginal costs contracts depends on the labour shares in the production of new houses - the higher the labour share the larger the fall in the marginal cost.

Since input prices inherit the stickiness of numeraire prices, firms reduce their demand for intermediate inputs, which results in a further contraction of numeraire output. The aggregate mark-up in the numeraire sector increases because prices fall less than the marginal cost. In the housing sector, the real marginal cost is not only a function of sectoral wages but also of adjustment cost in housing output. The extent to which the real marginal cost falls depends not only on the labour share in the sectoral production function but also on the parameter that controls the short-term price elasticity of housing supply. Due to the assumption that there is perfect competition in the housing sector, the fall in the marginal cost in the housing sector is passed directly onto house prices. Finally, figure 1 shows the results of our comparative analysis. We find that the responses of consumption, residential investment and aggregate output are more persistent in the IO relative to the other specifications. This result is consistent with Moro (2007).

Sensitivity analysis

In this section, we carry out a sensitivity analysis to check whether our results are robust. We look at a set of alternative policy rules and also vary the labour shares in the housing sector. To carry out the former, we follow the procedure adopted by Dhawan and Jeske (2007) to study the impact of monetary policy on the economic aggregates in our model. More precisely, we compare the performance of a baseline policy rule against and two alternative policy specifications to determine the success of the model in explaining comovement. We maintain the persistence of the interest rate persistence at $\mu_r = 0.7$ and change the weights on inflation and output fluctuations. Alternatively, we could assess optimal monetary policy rule but such exercise goes beyond the scope of our study.

We assess the following three policy rules:

1. **Rule 1 - Benchmark Rule** We use the Taylor rule as specified in the previous section:

$\mu_\pi = 0.5$ and $\mu_Y = 0$

2. **Rule 2 - Higher Weight on Inflation and Positive Weight on Output Fluctuations** We set the coefficient on inflation at $\mu_\pi = 1.5$ while keeping μ_Y at 0.13. and

3. **Rule 3 - High weight on Output Fluctuations and Lower Weight on Inflation** We increase the coefficient μ_Y to 0.26 and lower the inflation weight to $\mu_\pi = 0.35$.

Our results are robust to the three policy rules. Our results show that comovement arises independently of the weights given to inflation and output growth in the Taylor Rule. Figure 2 shows the responses of the main aggregates to a 1% increase in the nominal interest rate. We find that assigning more weight to inflation reduces the volatility of the main aggregates. This result follows from the fact that prices are sticky in the sector producing material inputs and consumption goods, so targeting inflation is consistent with the idea of Woodford (2003) that the monetary authority should implement rules that target the inflation of the prices with nominal rigidities.

We argue that, in a context with flexible consumer prices and sticky input prices, it would be optimal to target producer prices rather than consumer prices. The relative size of sectoral output marginally affects the volatility of output, so targeting output volatility yields much more volatile responses to the economic aggregates. If we were to increase the weight of the housing sector/durable sector, then targeting the volatility of output might not turn out to be a bad idea. We could alternative implement policies that target deviation of GDP from trend, but doing so marginally alters our results. In our model economy, a policy rule with a high weight on inflation has the potential to reduce the effect of price distortions. For our analysis, we choose the Taylor Rule that is consistent with CF.

We investigate the robustness of our results to changes in the labour share. We assess the performance of the model to monetary innovations for the following values of $\eta = \{0.42, 0.54, 0.66, 0.78, 0.9\}$. Figure 3 shows that the results are robust under all such parameters. We find that, even when we assume that the share of material inputs,$1-\eta = 0.1$, as in Iacoviello and Neri (2007), our model generates sectoral comovement. Therefore, we conclude that our results are highly robust.

4.2.2 Real Innovations

By introducing an IO structure into the two-sector Real Business Cycle (RBC) model with housing production, Davis and Heathcote (2005) argue that sectoral output comovement can be explained by a series of supply side shocks. The difference between their work and ours is that our model is a monetary one where the numeraire sector is the only producer of material inputs. Although our productive structure is simpler than theirs, our model delivers the correct comovement in output. Although we abstract from capital formation, the technological and government shocks produce the expected results.

Figure 4 shows the effect of a productivity shock affecting the numeraire sector. The higher productivity of labour together with the contraction in labour supply exerts upward pressure on the sectoral wage rate. The technological shock increases the supply of final goods and material inputs, putting downward pressure on the numeraire prices. On one hand, the real marginal cost in the numeraire sector falls due to the fact that the technology shock increases by more than wages. As a result, the sectoral mark-up increases. On the other hand, the real marginal cost in the housing sector is larger because of rising real wages and costly short-term adjustments in housing production. The representative firm in the housing sector passes the increase in the marginal cost directly onto house prices. Output comovement arises due to a sharp increase in the the demand for houses that stems from a higher wage bill. The consumer has the incentive to consume more consumption goods and more housing.

The technological shock in the housing sector does not have a large impact on the economy due to the fact that the housing sector is relatively small in comparison to the numeraire sector. As the shock hits the housing sector, labour demand increases, which puts upward pressure on the sectoral real wage. The change in the real marginal cost of the sector is insignificant because producing more housing is costly in the short-run due to the presence of the adjustment costs in the form of land. The higher demand for material inputs from the housing sector is offset by a fall in the demand for material inputs in the numeraire sector and by a relative small decline in consumption. Hence, numeraire prices react only slightly to the marginal fall in numeraire demand.

A temporary shock to government spending reduces the consumer's wealth through an increase in future taxation. Although the government can either raise more taxes or issue short-term debt to finance expenditure, over the long-term government bonds are assumed to be in zero net supply. The fall in the consumer's wealth reduces the demand for housing and consumption and it increases his/her supply of hours. As the contraction of consumption is lower than the increase in government expenditure, aggregate demand expands, putting upward pressure on numeraire prices. A higher demand for numeraire output increases the demand for intermediate inputs in that sector. The equilibrium effect on numeraire wages is indeterminate, as both demand and supply curves increase, but the numbers of numeraire hours is higher. Since the prices of material input are sticky in the short-term, more material inputs must be produced produced in equilibrium. The fact that more labour is supplied in equilibrium puts downward pressure on the real wage in the housing sector.

5 Nominal Wage Rigidities

A question that is valid to ask ourselves is: how do our results change when we allow for nominal wage rigidities? And in particular how does the real marginal costs vary in relation with benchmark case? The reason why it is interesting to ask such questions is to find whether results are complementary to the ones previously obtained. The first subsection introduces nominal wage rigidities into our model and the second presents the main results. To do so, we modify the problem of the consumer and compare the results against the benchmark IO model.

5.1 The Consumer and the Unions

Following a recent study by Schmitt-Grohe and Uribe (2006), we assume that firms in sector $s = \{n, h\}$ hire labour from two continua of labour markets of measure 1 indexed equally by $k \in [0, 1]$. Monopolistically competitive unions set wages in each market k and supply labour to satisfy the following individual demand

$$N_{st}(k) = \left[\frac{w_{st}(k)}{w_{st}}\right]^{-\breve{\epsilon}_s} N_{st}^d$$

where $\breve{\epsilon}_s$ is the wage elasticity of demand for each labour variety in sector s. The total amount of hours allocated to the labour markets must satisfy the following resource constraint

$$N_{st} = \int_0^1 N_{st}(k)\, dk \tag{36}$$

The representative consumer supplies each variety of labour $N_s(k)$ and receives a nominal wage $W_s(k)$ in return in sector s. We replace equation 1 with the following budget constraint

$$\begin{aligned} C_t + q_t\left[H_{ht} - (1-\delta_h) H_{ht-1}\right] + B_t \leq \frac{R_{t-1}}{\pi_t} B_{t-1} + \\ + N_{nt}^d \int_0^1 w_{nt}(k)\left[\frac{w_{nt}(k)}{w_{nt}}\right]^{-\breve{\theta}_m} dk + N_{ht}^d \int_0^1 w_{ht}(k)\left[\frac{w_{ht}(k)}{w_{ht}}\right]^{-\breve{\theta}_h} dk + \Phi_t + T_t \end{aligned} \tag{37}$$

where N_m^d and N_h^d denote the sectoral labour demands from the numeraire and housing sectors respectively. The first order conditions, equations (5) and (6), are replaced with

$$\Lambda_t \frac{w_{nt}}{\Upsilon_{nt}} = \chi\left(N_{n\tau}^{1+\xi} + N_{h\tau}^{1+\xi}\right)^{\frac{1+\varrho}{1+\xi}-1} N_{nt}^{\xi} \tag{38}$$

$$\Lambda_t \frac{w_{ht}}{\Upsilon_{ht}} = \chi\left(N_{n\tau}^{1+\xi} + N_{h\tau}^{1+\xi}\right)^{\frac{1+\varrho}{1+\xi}-1} N_{ht}^{\xi} \tag{39}$$

$$w_{nt}(k) = \begin{cases} w_{nt}^{*} & \text{if } w_{nt}(k) \text{ is set optimally in } t; \\ w_{nt-1}(k)/\pi_t & \text{otherwise.} \end{cases} \tag{40}$$

and

$$w_{ht}(k) = \begin{cases} w_{ht}^{*} & \text{if } w_{ht}(k) \text{ is set optimally in } t; \\ w_{ht-1}(k)/\pi_t & \text{otherwise.} \end{cases} \tag{41}$$

where Υ_s is the real wage mark-up that unions of type s impose on the each labour market. i.e. the mark-up is equal to the wedge between the disutility of labour and average real wage prevailing in sector s. The wage decision depends on the reset probability $\breve{\theta}_s$. If the central authority is not able to reoptimise the wage for v periods, the real wage prevailing in that market is $\tilde{w}_{st} \prod_{v=1}^{\tau} \left(\frac{1}{\pi_{t+v}}\right)$ and the labour demand $\left[\frac{\tilde{w}_{st}}{w_{s\tau}} \prod_{v=1}^{\tau} \left(\frac{1}{\pi_{t+v}}\right)\right]^{-\breve{\theta}_\tau} N^d_{s\tau}$. We omit from indexation of labour contracts are fully indexed if the labour unions is unable to reoptimise. The part of the Lagrangian that is relevant for the maximisation problem of the central authority is

$$\mathfrak{L}^w(s) = E_t \sum_{\tau=t}^{\infty} \left(\beta\breve{\theta}_s\right)^\tau \Lambda_\tau \left[\frac{\prod_{v=1}^{\tau}\left(\frac{1}{\pi_{t+v}}\right)}{w_{s\tau}}\right]^{-\breve{\epsilon}_s} N^d_{s\tau} \left\{ w_{st}^{1-\breve{\epsilon}_s} \prod_{v=1}^{\tau} \left(\frac{1}{\pi_{t+v}}\right) - w_{st}^{-\breve{\epsilon}_s} \frac{w_{s\tau}}{\Upsilon_{s\tau}} \right\}$$

After substituting in the above expression for the optimal choice of labour, it follows that the first order conditions with respect to w_{st} is

$$E_t \sum_{\tau=t}^{\infty} \left(\beta\breve{\theta}_s\right)^\tau \Lambda_\tau \left[\frac{w^*_{st}\prod_{v=1}^{\tau}\left(\frac{1}{\pi_{t+v}}\right)}{w_{s\tau}}\right]^{-\breve{\epsilon}_s} N^d_{s\tau} \left[\frac{(\breve{\epsilon}_s - 1)}{\breve{\epsilon}_s} w^*_{st} \prod_{v=1}^{s} \left(\frac{1}{\pi_{t+v}}\right) - \frac{w_{s\tau}}{\Upsilon_{s\tau}}\right] = 0 \tag{42}$$

This expression states that the optimal wage rate equates the central authority's future expected average marginal revenue to the average marginal cost of supplying labour[15]. From the nominal wage index, defined as $W_t = \left[\int_0^1 \left(W_t\left(k\right)\right)^{1-\breve{\epsilon}_s}\right]^{\frac{1}{1-\breve{\epsilon}_s}}$, it follows that the real wage evolves time according to

$$w_{st}^{1-\breve{\epsilon}_s} = \breve{\theta}_s \left(\frac{w_{st-1}}{\pi_t}\right)^{1-\breve{\epsilon}_s} + \left(1 - \breve{\theta}_s\right) \left(w^*_{st}\right)^{1-\breve{\epsilon}_s} \tag{43}$$

5.2 Further results

To simulate the Extended Input-Output (EIO) model with nominal wage stickiness, we calibrate the elasticity of substitution between sectoral labour in line with Schmitt-Grohe and Uribe (2006), $\breve{\epsilon}_s = 21$. We assume that unions operate equally in both sectors with the distinction that labour cannot be move freely across sectors. Consistent with the average reset time on prices in the baseline specification, we set $\breve{\theta}_n$ and $\breve{\theta}_h$ to 0.3. We maintain the other parameters as in 2.

[15]The recursive representation of the optimal pricing condition follows Schmitt-Grohe and Uribe (2004a)

Allowing for nominal wage rigidities conveys similar qualitative results to the ones of the baseline model with the addition that the EIO model helps alleviate the comovement problem in sectoral inputs. This finding further strengthens the results of the benchmark model in the sense that the correct response of sectoral labour is independent the source of the shock. In particular, the response of labour to a monetary innovation is negative because wages cannot adjust freely, so the incentive of the consumer to move labour towards the numeraire sector is further dampened. Secondly, we find that the dynamics of the model does not vary qualitatively a great deal with the introduction of nominal wage rigidities. As the shares of labour in the production functions are relatively small in the EIO model, the real marginal cost in both sectors is not significantly affected. Thus, we find that our solution can be used as alternative or complementary to previous analysis and that varying the productive structure strengthens the solution of the puzzle.

We conjecture that the reason behind the qualitatively similar responses to a government and productivity shocks is related to the productive structure of the model. Labour in the numeraire sector contracts due to the fact that the introduction of the nominal wage rigidities increase the relative demand for material inputs. The responses of the economy to productivity shocks exhibit higher volatility when we allow for nominal wage rigidities. The underlying reason is clear: the introduction of nominal wage rigidities increases the rigidity of the real marginal costs in both sectors, which in turn reduces the volatility of prices but increases volatility of sectoral output. In the model with an IO structure, the source of the shock has asymmetric responses on the variability of the economic aggregates.

The real marginal cost of the housing sector inherits both the rigidities of nominal wages and input prices. In addition, it would be more appropriate to compare the benchmark model with the extension version of the model using optimal rules because the model dynamics is highly dependant on the choice of such rules. To improve the cross-comparison of the models presented here, future work could assess optimal monetary rule in line with the work by Erceg, Henderson, and Levin (2000), Schmitt-Grohe and Uribe (2004a) and Monacelli (2006) and others.

6 Conclusion

We find that the introduction an Input-Output structure to the standard two-sector New Keynesian Model with housing production can be used as an alternative/complementary avenue to solve the comovement puzzle originating from a monetary innovation. We argue that in an production economy with an Input-Output structure the productivity and government expenditure shocks also generate the expected results. By comparing the model with other model specifications, that are embedded in our model, we find that the response of the economic aggregates to shocks are in accordance with empirical evidence. We show that the results are robust under alternative policy rules and under different values of the labour share in the production of new houses. The introduction of nominal wage rigidities reduces the comovement puzzle in sectoral inputs.

Future work could investigate the dynamics of the model when we allow for capital formation. We postulate however that introducing sector specific capital should not alter the results significantly as the share of capital in the production of new houses is much lower than the share in the numeraire sector. We argue that the combination of a more flexible material input aggregator with sector-specific capital could have an impact on the dynamic implications of the model. We could drop the assumption that the elasticity of substitution between variety of material inputs is the same across sectors or assume that the representative firm in the housing sector produces own material inputs. All such extensions however present additional complications that are unnecessary for providing an alternative solution to the puzzle. A more attractive avenue of research would be to assess optimal monetary policy in a two-sector economy with housing and an IO structure. Most importantly, as we have worked under the assumption of perfect capital markets, an additional extension of our analysis would be to introduce collateralised borrowing and lending.

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Appendix

A From Housing to Durables

In this appendix we generalise our model to durable consumption. To do so, we introduce an alternative specification for the durable production function in order to make the model independent of land. We maintain the assumption that durable prices are flexible relative to non-durable prices. Although the evidence points towards the fact that durable prices are stickier than housing and that the stickiness of durable prices is generally lower than that of non-durables. We could introduce price rigidity in the durable sector following Monacelli (2008), in which case the interest rate rule must be modified to account for these new rigidities. However, we conjecture that doing so should not alter the main results of our analysis. The findings of this appendix are qualitatively similar to the ones obtained in the main text.

A.1 The Durable Sector

The model economy has now two sectors, one producing non-durables final goods and material inputs and the other producing durable goods. Although the numeraire sector is similar to the one stated in the main text, the durable producing sector abstracts from land as a input for production. In particular, the durable producing sector features an adjustment cost of the form

$$Y_{dt} \leq Y_{dt}^{l} \left[1 - \frac{\varphi}{2} \left(\frac{Y_{dt}}{Y_{dt-1}} - 1 \right)^{2} \right] \tag{44}$$

This specification of the adjustment cost function follows the one proposed by Ireland and Schuh (2006) in that changes in short-term output are costly. The long-term production function of durables, Y_{dt}^{l}, is a constant return to scale technology that uses both sectoral labour, N_{dt}, and material inputs, M_{dt}. The functional form of such function is given by

$$Y_{dt}^{l} = Z_{dt} N_{dt}^{\eta} M_{dt}^{1-\eta}$$

with

$$Z_{dt} = \rho_h Z_{dt-1} + \varepsilon_{dt}, \quad \varepsilon_{dt} \sim \mathbf{N}\left(0, \sigma_h^2\right) \tag{45}$$

When maximising profits, the durable producing firm takes input prices, w_{dt}, as given to solve the following maximisation problem

$$\max_{N_{dt}, M_{dt}} \quad p_{dt} Y_{dt}^{s} - w_{dt} N_{dt} - M_{dt}$$

subject to equation (44). Due to the fact that adjustments to the level of durable production are costly, the constrained Lagrangian of the the representative firm in the housing sector is given by

$$\begin{aligned}\mathcal{L}_f^h &= E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{\Lambda_\tau}{\Lambda_t} \{ q_{d\tau} Y_{d\tau} - w_{d\tau} N_{d\tau} - M_{d\tau} + \\ &+ \Psi_\tau \left[Z_{d\tau} N_{d\tau}^{\eta} M_{d\tau}^{1-\eta} \left[1 - \frac{\varphi}{2} \left(\frac{Y_{d\tau}}{Y_{d\tau-1}} - 1 \right)^2 \right] - Y_{d\tau} \right] \right\}\end{aligned}$$

The first order conditions with respect to N_{dt}, M_{dt} and Y_{dt} are

$$\eta \Psi_t \frac{Y_{dt}}{N_{dt}} = w_{dt} \tag{46}$$

$$(1-\eta) \Psi_t \frac{Y_{dt}}{M_{dt}} = 1 \tag{47}$$

and

$$q_{dt} = \Psi_t \left[1 + \varphi \left(\frac{Y_{dt}}{Y_{dt-1}} - 1 \right) \frac{Y_{dt}^l}{Y_{dt-1}} \right] - \beta \varphi E_t \frac{\Lambda_{t+1}}{\Lambda_t} \Psi_{t+1} \left(\frac{Y_{dt+1}}{Y_{dt}} - 1 \right) \frac{Y_{ht+1}^l Y_{dt+1}}{(Y_{dt})^2} \tag{48}$$

where Ψ_t is the Lagrange multiplier corresponding the maximisation problem of the representative firm in the housing sector. By combining equations (44),(46) and (47), we derive an expression for the multiplier

$$\Psi_t = \left[\frac{1}{A_{dt}} w_{dt}^{\eta} \frac{1}{(1-\eta)^{1-\eta} \alpha^{\eta}} \right] / \left[1 - \frac{\varphi}{2} \left(\frac{Y_{dt}^s}{Y_{dt-1}^s} - 1 \right)^2 \right]$$

In the deterministic steady state, the value of the multiplier is equal to the first part of the expression above due to the fact that the second part of the expression cancels in the steady state. Hence, the value of q_{dt} and Ψ_t in the steady state are both equal to the real marginal cost in the housing sector.

We log-linearise expression (48) to recover an expression for the short-term price elasticity of the durable supply. The linearised version of equation (48) is

$$\widehat{Y}_{dt}^s - \widehat{Y}_{dt-1}^s = \frac{p_{dt} - q_{dt}}{\varphi} + \beta \left(\widehat{Y}_{dt+1}^s - \widehat{Y}_{dt}^s \right) \tag{49}$$

where the variable with a hat represent the percentage deviation of the variable with respect to the steady state. By iterating this expression forward we get the following expression

$$\widehat{Y}_{dt}^s = \frac{1}{\varphi} \sum_{\tau=t}^{\infty} \beta^{\tau-t} (p_{h\tau} - q_{h\tau}) + \widehat{Y}_{dt-1}^s \tag{50}$$

It follows that the short-term price elasticity of housing supply is $1/\varphi$[16].

[16] We thank Timothy Fuerst for clarification on this point.

A.2 The Representative Consumer

The consumer problem is different from the model presented in the main text in that his or her optimisation problem abstracts from land. The other difference is the redefinition of the variable D_t, that amounts for the level of the durable stock. The representative consumer's period utility is

$$U\left(X_t, N_t\right) = \left(\frac{X_\tau^{1-\sigma} - 1}{1-\sigma} - \chi \frac{N_\tau^{1+\varrho}}{1+\varrho}\right)$$

Note that the consumption aggregator is $X_t = \left(C_t - bC_{t-1}\right)^{\zeta}\left(\kappa D_t\right)^{1-\zeta}$ and the labour aggregator is $N_t = \left(N_{nt}^{1+\xi} + N_{dt}^{1+\xi}\right)^{\frac{1}{1+\xi}}$. Therefore, the representative consumer maximises his or her lifetime utility subject to a infinite sequence of budget constraints of the form

$$C_t + p_{dt}\left[D_t - \left(1-\delta_h\right)D_{t-1}\right] + B_t \leq \frac{R_{t-1}}{\pi_t}B_{t-1} + w_{nt}N_{nt} + w_{dt}N_{dt} + \Phi_t + T_t \tag{51}$$

The constrained Lagrangian for the consumer is given by

$$\begin{aligned}\mathcal{L}_h^{nw} = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Bigg\{ & \frac{\left[\left(C_\tau - bC_{\tau-1}\right)^{\zeta}\left(D_\tau\right)^{1-\zeta}\right]^{1-\sigma}}{1-\sigma} - \chi \frac{\left(N_{n\tau}^{1+\xi} + N_{h\tau}^{1+\xi}\right)^{\frac{1+\varrho}{1+\xi}}}{1+\varrho} + \\ & + \Lambda_\tau \left[\frac{R_{\tau-1}}{\pi_\tau}B_{\tau-1} + w_{d\tau}N_{d\tau} + w_{n\tau}N_{n\tau} + \Phi_t + T_t - C_\tau - q_\tau\left[D_\tau\left(1-\delta_d\right)D_{\tau-1}\right] - B_\tau\right]\Bigg\}\end{aligned}$$

where $\beta^t\Lambda_t$ is the Lagrange multipliers associated with the consumer's budget constraints (51).

A stationary competitive equilibrium is a set of endogenous stationary processes, Λ_t, C_t, D_t, N_{nt}, N_{ht}, R_t, Π_t, q_{ht}, r_{lt}, W_{nt}, W_{ht}, M_{nt}, M_{dt}, Y_{nt}, mc_{nt}, Y_{dt}, $\tilde{P}_t$ and exogeneous stochastic process $\left\{Z_{nt}, Z_{ht}, Z_{mt}, G_t\right\}_{t=0}^{\infty}$ satisfying (2)-(6), (8)-(14), (44)-(48),(22)-(23) given the initial conditions for C_{-1}, H_{-1}, R_{-1}, $Y_{n,-1}$. Finally, the computation of the steady state is similar to the one stated in the main text with the exception that no need for normalisation of the land stock is needed. It is worth noting that the model must be calibrated differently from the model with housing.

B Normalising the Value of the Land Sock

In this appendix, we show that the simulations are homogeneous irrespective of the choice of normalisation for the stock of land. On one hand, the normalisation of land to value of residential investment, $L = \bar{Y}_h$, generates a production function that exhibits constant returns to scale in the steady state. One the other, normalising the value of land to 1 results in production of new houses that exhibits decreasing returns to scale in the steady state. The question that we pose in this Appendix is: Does this normalisation alter the dynamic properties of the model?

The answer to this question is no because the dynamic relationships between the economic variables remain unaltered. All the equilibrium conditions remain unchanged with the only difference being the values of the main aggregates at the steady state. Hence, the results of the simulations are substantially the same. However, normalising land to a value that is different from Y_h presents an additional complication. This complication is a computational one as the the steady state can no longer be solved analytically. To solve for the steady state, we reduce the system of equations to 5. The five variable of interest after substituting for some economic relationships are residential investment, Y_n, house prices, Q_h, residential investment, Y_h, aggregate output, Y and the aggregate real wage rate, w, defined as $w = \left(w_n^{1+\varrho} + w_h^{1+\varrho}\right)^{1/1+\varrho}$. As the system is non-linear, the steady state must be solved using non-linear techniques. We implement the Gauss-Newton Method to solve for the values of these five variables. We choose the starting value of 1 for all remaining variables. We use the fsolve command of MATLAB to solve for the variables.

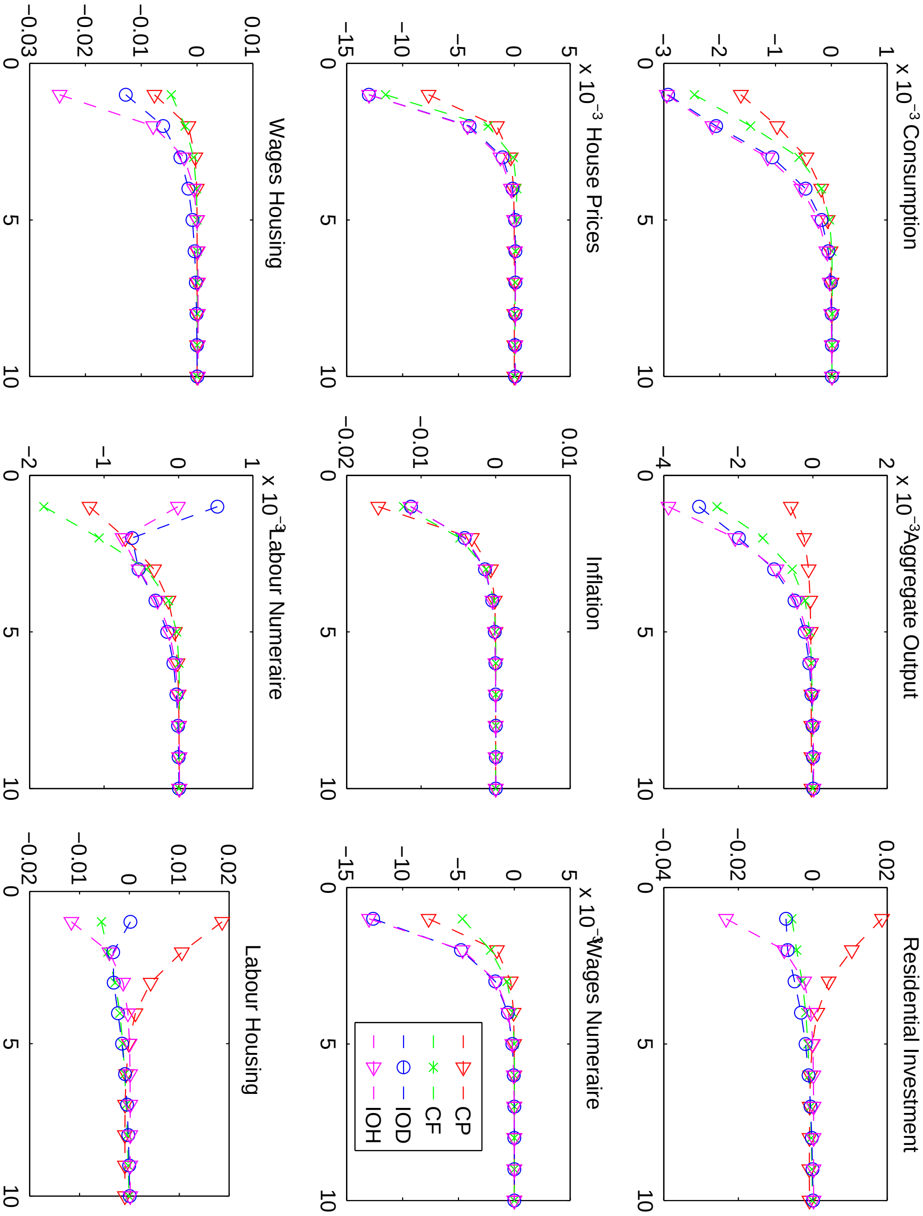

Figure 1: Responses of main aggregates to a monetary shock

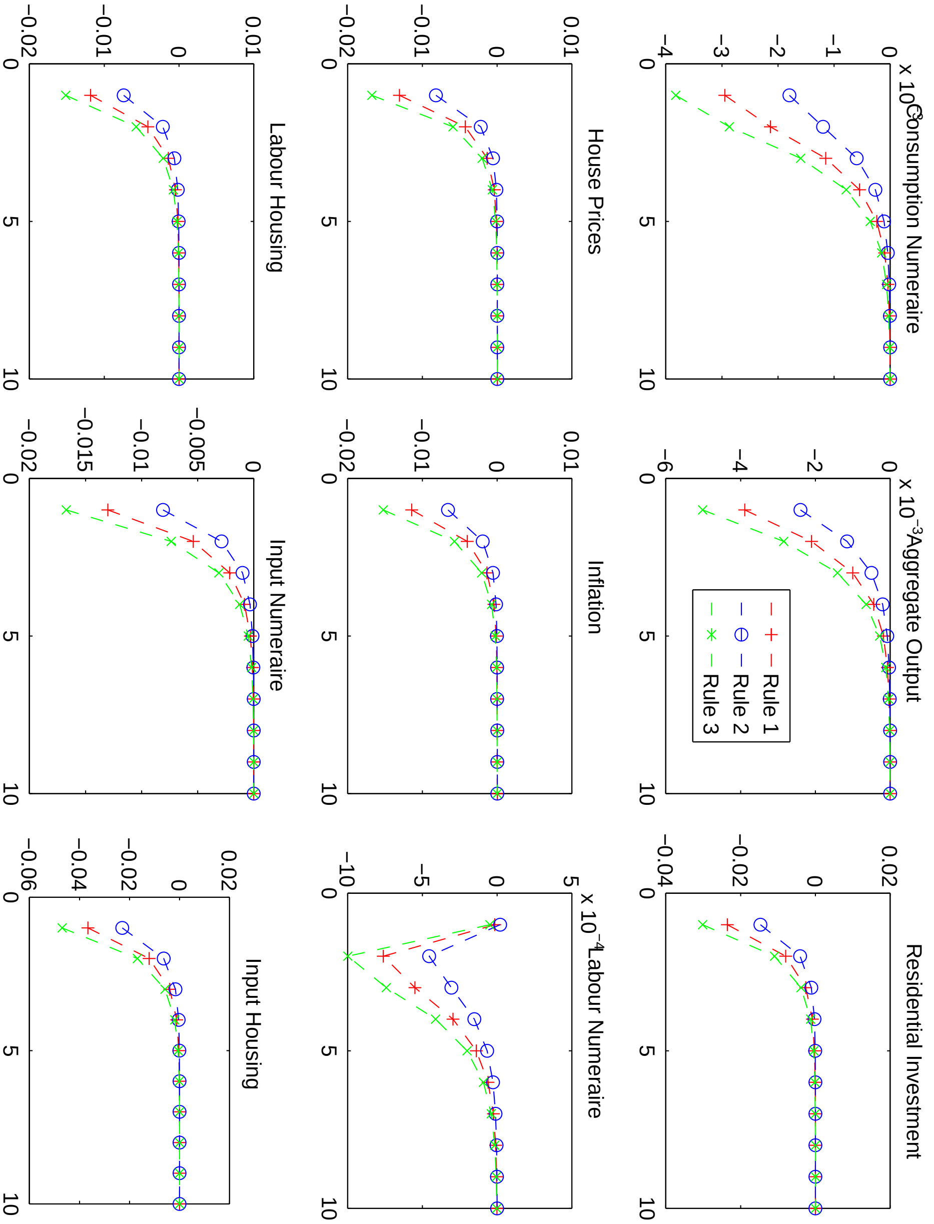

Figure 2: Impulse responses to a monetary shock under different policy rules

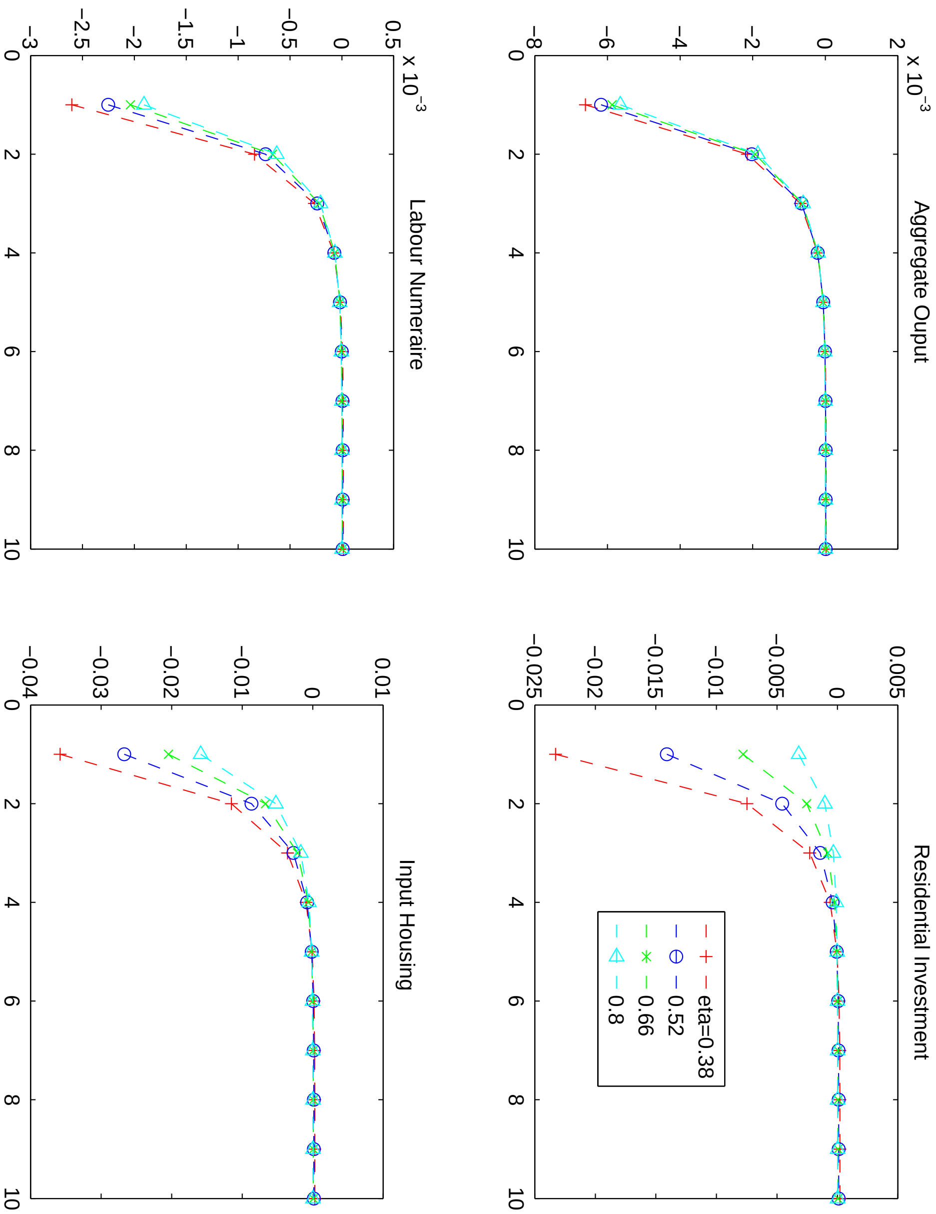

Figure 3: A monetary shock under different values of η

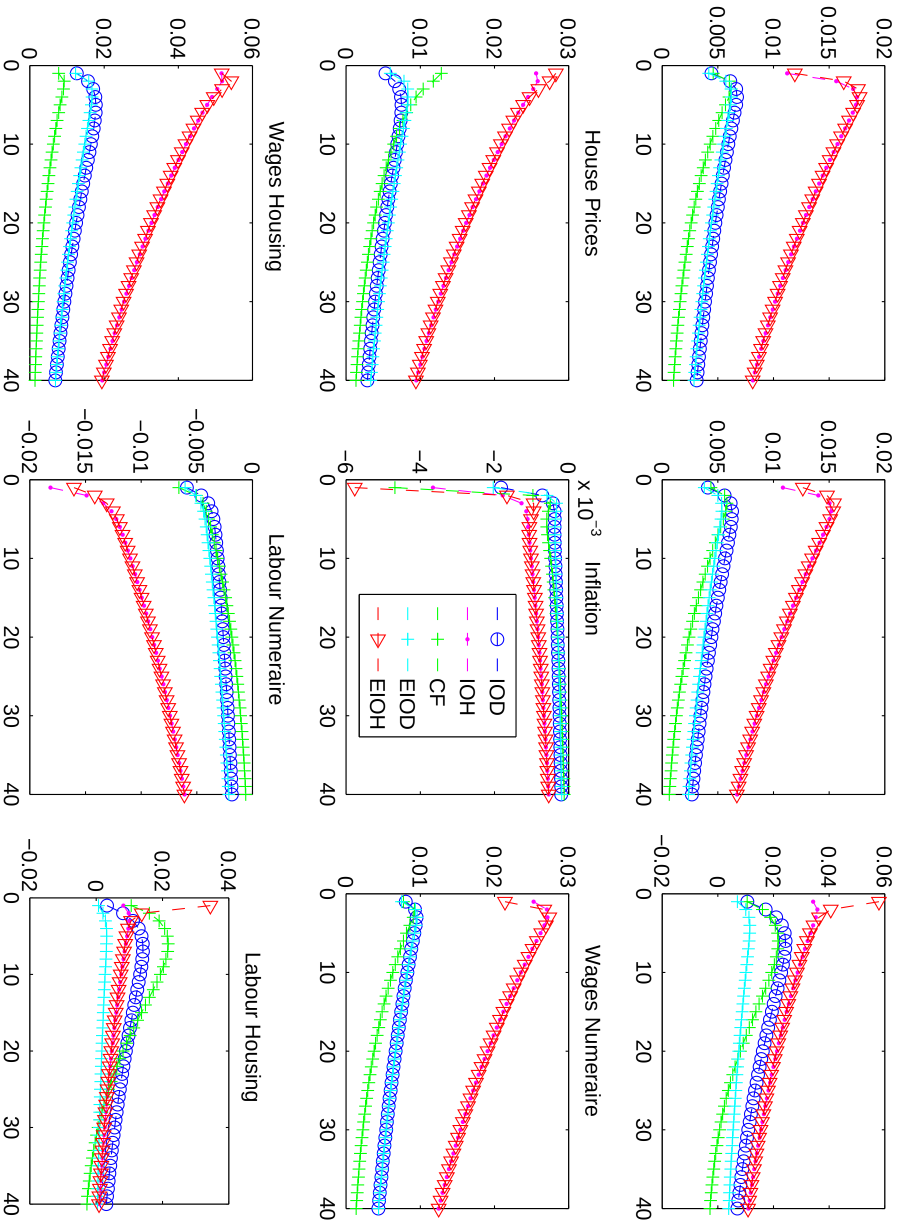

Figure 4: Impulse responses to a productivity shock in the numeraire sector

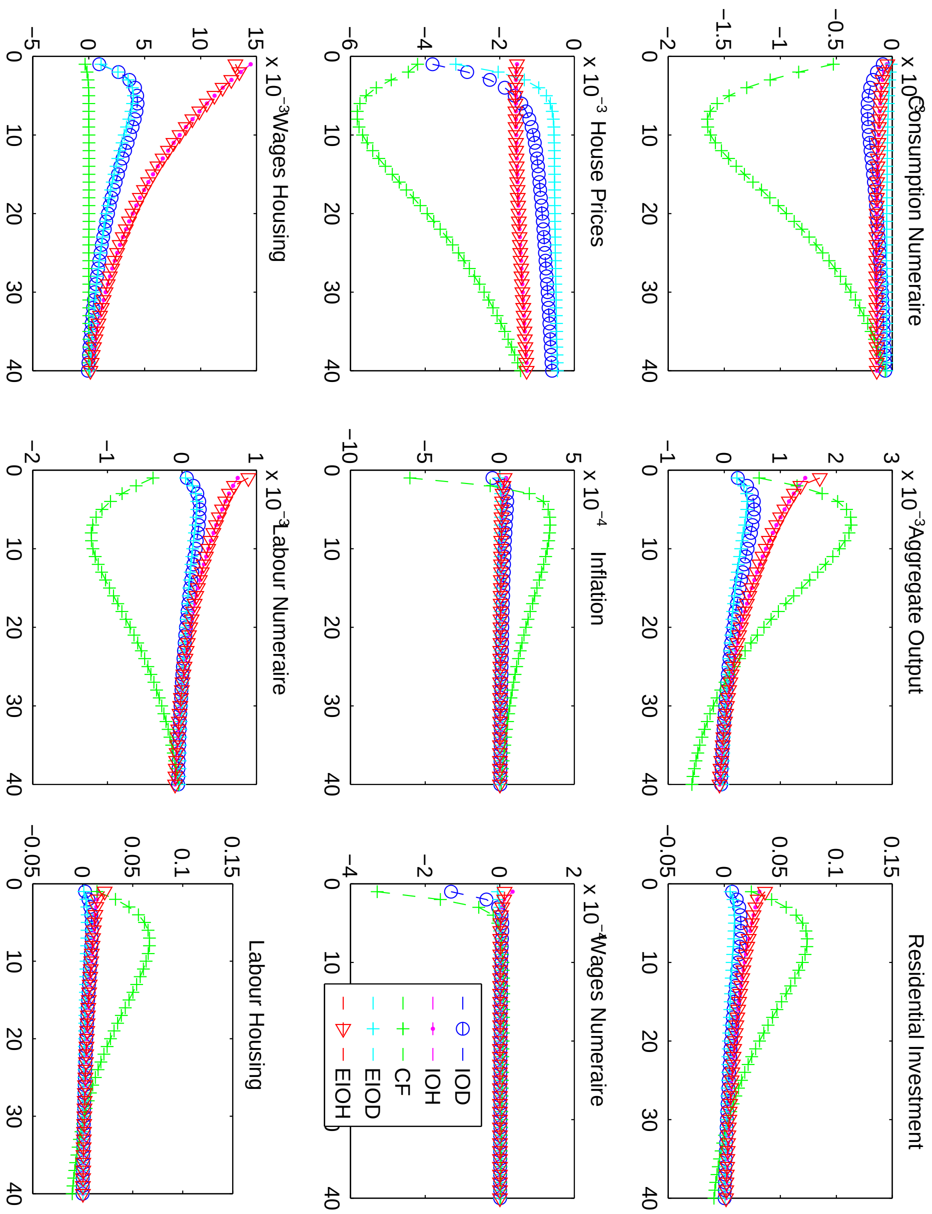

Figure 5: Impulse responses to a housing productivity shock

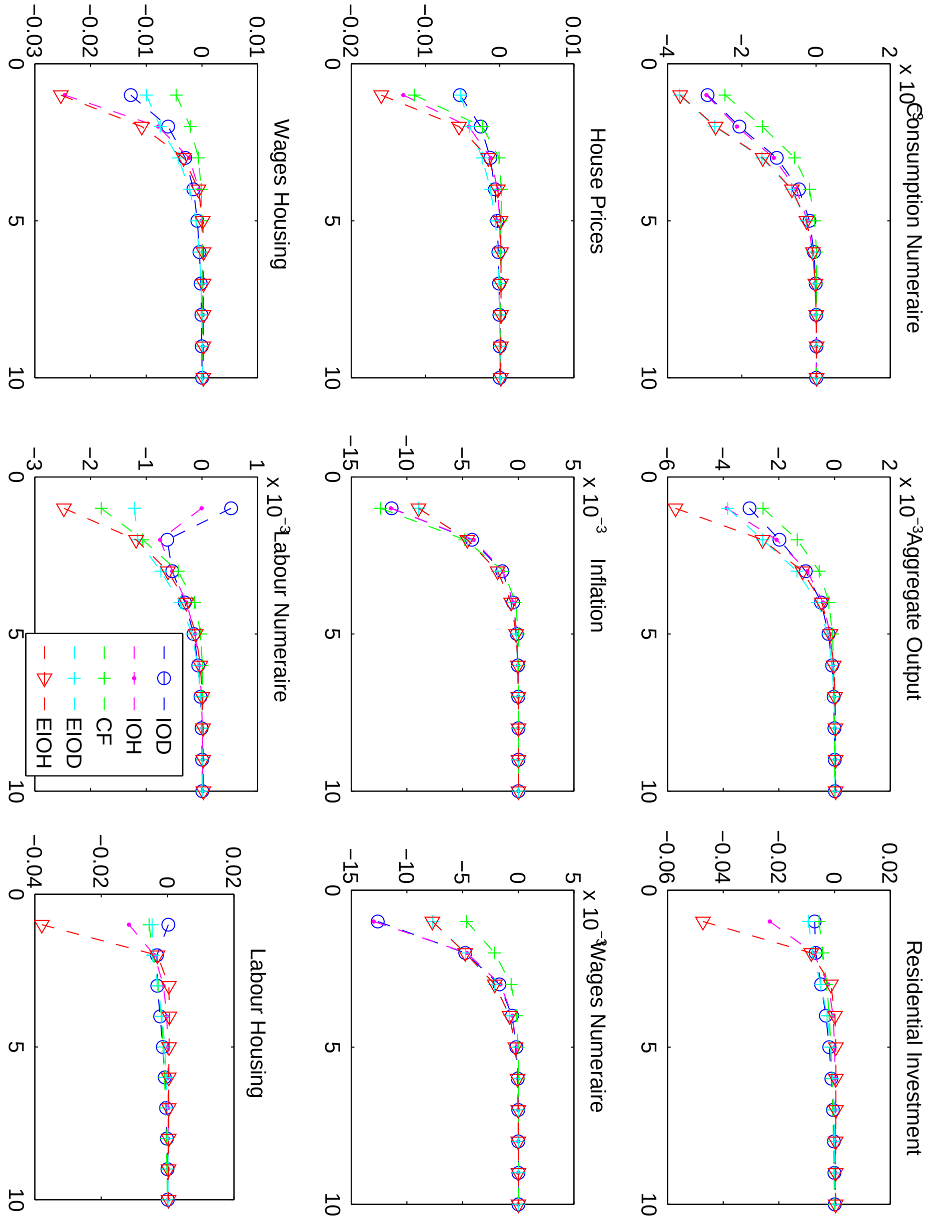

Figure 6: Impulse responses to a monetary shock