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Abstract

This article analyzes promotion tournaments where candidates engage in multiple tasks. We consider a promotion rule where the winner of the promotion tournament is randomly selected from the best performers at each task. The promotion tournament can achieve an efficient outcome for any production uncertainty (observability) of tasks and substitutability in the effort cost when employees are risk neutral and homogeneous. The promotion decision should be based much more on the outcome in a more uncertain task. If employees are heterogeneous in their ability to undertake a task, then the outcome of an ability-dependent task should be relied upon more in the promotion decision than the outcome of a simple task.

Keywords: Tournament; Multitask agency; Promotion

JEL classification: D82; M51

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1 Introduction

Employees in organizations normally engage in jobs that contain multiple tasks, or in other words, the performance of their job is multidimensional. For example, production workers are required to care about quantity and quality, and professors have to carry out research as well as teach. On the other hand, it is well known that using promotions is important inside organizations to provide employees with incentives[1]. However, it is difficult to decide which employees should be promoted when employees engage in multiple tasks. For example, if employee i is superior at task a (quantity) but employee j is superior at task b (quality), which employee should obtain a promotion? If the winner is always chosen based on the best performance of task a, then employees will concentrate their effort on performing well in task a.

A typical promotion model is a tournament model, but this has not been used to examine such an incentive allocation problem with multiple tasks. Dye (1984) pointed out some limitations of tournaments, and commented, 'When outputs are multidimensional, to designate who has done the better job requires a comparison of (multidimensional) vectors of outputs, and there is no intrinsic reason to believe that such vector rankings are simpler to make than performance-contingent wage assignments' (p. 148). Therefore it is important to develop a tournament model into a multi-task setting. The purpose of this paper is to study promotion tournaments where employees engage in multiple tasks, and to investigate the optimal promotion rule to ensure an efficient incentive structure.

Holmstrom and Milgrom (1991) first analyzed multitask problems with linear compensation contracts[2]. The simplified version of the linear compensation considered by these authors is as follows. Suppose that employees undertake tasks a and b, and that their measurable output is given by $y_t = e_t + \epsilon_t$ $(t = a, b)$, where e_t is the effort and ϵ_t are normally distributed measurement errors with a variance σ_t^2. The linear wage scheme is given by $w(y) = \alpha + \beta_a y_a + \beta_b y_b$, where β_a and β_b represent the intensity of incentives for task a and task b, respectively. The two types of effort are assumed to be substitutes in the employees' cost function. Then, Holmstrom and Milgrom

[1] There are theoretical works explaining why organizations use promotions to motivate employees. Prendergast (1993) investigated the case where firm-specific human capital investment is unverifiable, and Fairburn and Malcomeson (2001) analyzed the situation where employees can influence a manager's report using bribes. Both papers showed that promotion can provide incentives to employees, but also that performance-related pay does not work.

[2] There have been some papers that have analyzed an optimal compensation scheme when employees engaged in multiple tasks. For example, see Itoh (1991, 1994), Meyer et al. (1996) and MacDonald and Marx (2001).

showed that the fixed wage ($\beta_a = \beta_b = 0$) could be optimal when the output of task t is difficult to measure ($\sigma_t^2 \to \infty$).

No incentives, or very low incentives, can be optimal in the linear compensation model because of the effort substitution and the difference between the observability of tasks. Therefore, Holmstrom and Milgrom (1991) pointed out that the agents should be given a single task, or else the principal should design the job based on the observability of tasks. In contrast to the static multitask model of Holmstrom and Milgrom (1991), Meyer et al. (1996) analyzed a dynamic model with a limited commitment. They showed that broad task assignments (joint responsibility) and heterogeneous grouping of tasks with respect to the ease of measuring performance are optimal when the ratchet problem is severe [3].

Although designing a job is an important component of mitigating incentive problems, regrouping tasks based on observability is sometimes not possible, or it induces a loss of efficiency in the production process. Lazear (1995) argued that, 'Technology is likely to be much more important than the incentive issue. For example, professors teach and do research on the same subject not because the output of each is observable but because the production of research and teaching are complementary' (p. 87). Therefore, in this paper, we do not consider job design, but instead focus on the promotion tournament, given a job that contains two tasks. This paper considers the optimal promotion rule to motivate employees to exert effort in two tasks for any level of performance observability.

The rule analyzed in this paper is that the winner is randomly selected from among the best performers at each task. The employer determines the probability and the wage increase at promotion ex ante. In this paper, the probability of performance in a particular task being used in the promotion decision determines the allocation of effort between the two tasks and the wage increase determines the strength of the incentive.

The main results are summarized as follows. The promotion tournament can achieve an efficient outcome for any production uncertainty (observability) of tasks and substitutability in the effort cost when employees are risk neutral and homogeneous. Therefore, there is no need to regroup tasks and the limitations of a tournament can be overcome. Promotion decisions should be based more heavily on the outcome in a more uncertain task. If employees are heterogeneous in their ability to undertake one of the two tasks involved in a job, then the information from an ability-dependent task should be relied

[3] Schottner (2007) analyzed the job design problem in relational contracts with risk-neutral agents. He showed that cases exist where broad task assignments are optimal when the imperfect performance measure strongly distorts incentives for splitting tasks and when broader task assignments improve the implicit incentives.

upon more in the promotion decision than in a simple task.

The rest of the paper is organized as follows. In the next section, the model is presented. We develop the model by Lazear and Rosen (1981) for the case of multiple tasks. In Section 3, we solve the promotion tournament and show the optimal promotion rule. In Section 4, we analyze the promotion tournament with heterogeneous employees, and Section 5 concludes the paper.

2 The Model

We consider a promotion tournament with two identical candidates, i and j. They engage in a job that involves two tasks, a and b. The employee, k $(k = i, j)$, privately chooses his or her effort, (e_a^k, e_b^k), where e_t^k is the effort supplied to task t $(t = a, b)$. The personal cost is given by $C(e_a^k, e_b^k)$. We assume that the cost function is strictly increasing and strictly convex. For analytical simplicity, the cost function takes the following quadratic form:

$$C(e_a^k, e_b^k) = \frac{1}{2}{e_a^k}^2 + \frac{1}{2}{e_b^k}^2 + \delta e_a^k e_b^k,$$

where $\delta \in [0, 1)$ represents the degree of cost substitutability between the two tasks.

The employer cannot observe an employee's effort (or the employee's true contribution to the firm) directly, but he or she can obtain some information from each task. The observed measures of employer k in task t $(t = a, b)$ are given by:

$$q_t^k = e_t^k + \epsilon_t^k,$$

where $\epsilon_t^k (t = a, b)$ is a measurement error that is normally distributed with $E(\epsilon_t^k) = 0$ and $E({\epsilon_t^k}^2) = \sigma_t^2$. The term ϵ_t^k is independently and identically distributed across the employees. In addition, we assume that ϵ_a^k and ϵ_b^k are independently distributed, $(\sigma_{ab} = 0)$. When $\sigma_a^2 > \sigma_b^2$, it means that task a involves more uncertainty for an employee than task b does.

The employer must decide which employee is to be promoted. The employer can attach a wage to a job. The promoted employee receives a wage, W_h, whereas the wage of the employee who is not promoted is W_ℓ, where $W_h > W_\ell$. We denote the wage increase that occurs on promotion as $\Delta W = W_h - W_\ell$. Note that wages are determined completely by the jobs, and that the winner's wage does not depend on whether he or she is superior in both tasks or in only one. There are two reasons why we do not consider the wage scheme in which an employee is paid W_h when he or she is superior in both tasks and $W_h'(< W_h)$ when an employee is superior in only one task and becomes the winner of the promotion tournament. The ranking of performance in each task should be verified to provide incentives to use such a wage scheme because the employer has an incentive to cheat and announce that the winner is superior in only one job. Even if the ranking of performance is verifiable and it is possible to design such a wage scheme, the employer cannot improve upon the result, because the job-related wage scheme implements the first-best solution, as we will show in the next section.

The expected utility for employee k $(k = i, j)$ is given by:

$$\begin{aligned} U^k &= P^k W_h + (1 - P^k) W_\ell \\ &= W_\ell + P^k \Delta W - C(e_a^k, e_b^k), \end{aligned}$$

where P^k is the probability that employee k wins the promotion competition.

It is not possible to engage in tasks a and b separately and the risk-neutral employer receives a payoff of $B(e_a^i + e_a^j, e_a^i + e_b^j) - W_h - W_\ell$, where $B(e_a^i + e_a^j, e_a^i + e_b^j)$ represents the benefit from the two tasks. $B(\cdot)$ is assumed to be strictly increasing and concave. We assume that $B_{ab} \geq 0$. This assumption means that both tasks are complements in the production technology, whereas, as mentioned above, there is an effort substitution in the employees' cost function.

The promotion decision is easy if a contestant is superior to another contestant in both tasks. However, there is a high chance that an employee i is superior to employee j in task a but inferior in task b. Therefore, the employer should set the rule of promotion ex ante to provide incentives for employees to make efforts in both tasks. We consider the following rule.

The promotion rule: The best performer in task a obtains a promotion with probability x, and the best performer in task b obtains a promotion with probability $1 - x$.

The employer chooses the probability x, and the wages W_h and W_ℓ before the employees make their efforts. Employees know the rule, and make a decision as to how much effort to expend on each task.

This rule of promotion can be interpreted as follows. Suppose that there are two supervisors. One observes the employees' performance at task a and the other observes their performance at task b. For example, one supervisor is responsible for the quality of a product, whereas the other is responsible for the quantity. Both supervisors report to the employer which employee is superior in performing the task on which they check. Then, the employer plays a mixed strategy, and adopts one opinion with probability x. Alternatively, we can interpret the promotion rule as follows. Suppose that the employer monitors the performance of only one task. The probability that he or she chooses to monitor task a is x, ex ante. In this case, the employer does not need to monitor the performance of both tasks, and can save on monitoring costs. Therefore, this promotion rule has another advantage.

3 Results

First, we consider the employee's decision about how much effort to expend on each task, given the wage scheme and the promotion rule. Given the promotion rule x, the probability that employee i defeats employee j is:

$$\begin{aligned} P^i &= x \Pr(y_a^i > y_a^j) + (1-x) \Pr(y_b^i > y_b^j) \\ &= x \Pr(e_a^i - e_a^j > \epsilon_a) + (1-x) \Pr(e_b^i - e_b^j > \epsilon_b) \\ &= x G_a(e_a^i - e_a^j) + (1-x) G_b(e_b^i - e_b^j), \end{aligned}$$

where $\epsilon_t \equiv \epsilon_t^j - \epsilon_t^i$ and $G_t(\cdot)$ is its cumulative distribution $(t = a, b)$. As ϵ_t^i and ϵ_t^j are independent random variables and normally distributed with mean zero and variance, σ_t^2, then ϵ_t is normally distributed with mean zero and variance, $2\sigma_t^2$.

Given employee i's conjectured effort level regarding his or her opponent j, employee i chooses e_a^i and e_b^i to maximize his or her expected utility. The expected utility of employee i can be rewritten as:

$$U^i = W_\ell + \Delta W \left\{ x G_a(e_a^i - e_a^j) + (1-x) G_b(e_b^i - e_b^j) \right\} - C(e_a^i, e_b^i).$$

Employee i's strategy is characterized by the first-order conditions:

$$\begin{aligned} x g_a(e_a^i - e_a^j) \Delta W &= C_a(e_a^i, e_b^i) \\ (1-x) g_b(e_b^i - e_b^j) \Delta W &= C_b(e_a^i, e_b^i), \end{aligned}$$

where $g_t(\cdot)$ is the density function of ϵ_t. Employee j solves the corresponding problem.

Lemma 1. *An asymmetric equilibrium does not exist where employee i chooses $(e_a^i, 0)$ and j chooses $(0, e_b^j)$.*

Proof. Suppose that employee i chooses $(e_a^i, 0)$ and employee j chooses $(0, e_b^j)$. The first-order conditions for employee i imply that:

$$x g_a(e_a^i) \Delta W = C_a(e_a^i, 0) > C_b(e_a^i, 0) = (1-x) g_b(-e_b^j) \Delta W.$$

The first-order conditions for employee j imply that:

$$x g_a(-e_a^i) \Delta W = C_a(0, e_b^j) < C_b(0, e_b^j) = (1-x) g_b(e_b^j) \Delta W.$$

Since $g_t(\cdot)(t = a, b)$ is symmetrical at zero, we obtain a contradiction. $\square$

We focus on a symmetric equilibrium, such that $e_a^i = e_a^j \equiv e_a^*, e_b^i = e_b^j \equiv e_b^*$. Then, the first-order conditions reduce to:

$$\begin{aligned} x g_a(0)\Delta W &= C_a(e_a^*, e_b^*) && (1) \\ (1-x) g_b(0)\Delta W &= C_b(e_a^*, e_b^*). && (2) \end{aligned}$$

The second-order conditions are:

$$U_{aa} < 0, \qquad U_{bb} < 0, \qquad \begin{vmatrix} U_{aa} & U_{ab} \\ U_{ba} & U_{bb} \end{vmatrix} > 0.$$

The convexity of $C(e_a, e_b)$ implies that the second-order conditions are satisfied at $(e_a^*,\ e_b^*)$.

Inserting the normal density and the quadratic specification of $C(\cdot)$, equations (1) and (2) become, respectively:

$$\begin{aligned} x\frac{\Delta W}{2\sigma_a\sqrt{\pi}} &= e_a^* + \delta e_b^* && (3) \\ (1-x)\frac{\Delta W}{2\sigma_b\sqrt{\pi}} &= e_b^* + \delta e_a^*. && (4) \end{aligned}$$

The equilibrium effort for each job is given by:

$$\begin{aligned} e_a^* &= \frac{\Delta W}{(1-\delta^2)2\sqrt{\pi}}\left(\frac{x}{\sigma_a} - \frac{\delta(1-x)}{\sigma_b}\right), && (5) \\ e_b^* &= \frac{\Delta W}{(1-\delta^2)2\sqrt{\pi}}\left(\frac{1-x}{\sigma_b} - \frac{\delta x}{\sigma_a}\right). && (6) \end{aligned}$$

Thus, we obtain the next lemma.

Lemma 2. *(a) The level of effort expended on task a increases, but the level of effort expended on task b decreases with the probability of judging a promotion by the outcome of task a.*
(b) The level of effort on both tasks increases with a wage increase on promotion.

Both the wage increase and the assessment probability affect an employee's strategy. Lemma 2 implies that ΔW determines the strength of an incentive, which is the same as in the traditional tournament model, and x determines the allocation of effort.

The employee's individual rationality constraint is:

$$W_\ell + \frac{1}{2}\Delta W - C(e_a^*, e_b^*) \geq \bar{U}, \qquad (7)$$

where $\bar{U}$ is the employee's reservation utility.

Given the employee's strategy described in (5) and (6), and the individual rationality constraint (7), the employer designs W_ℓ, ΔW and x to maximize his or her expected profit.

$$B(2e_a^*, 2e_b^*) - (2W_\ell + \Delta W) \tag{8}$$

On the other hand, the first-best effort, (e_a^{FB}, e_b^{FB}), is the solution of the following problem:

$$\max_{e_a, e_b} \qquad B(2e_a, 2e_b) - 2(\bar{U} + C(e_a, e_b)).$$

Thus, the first-best effort, (e_a^{FB}, e_b^{FB}), is characterized by the following first-order conditions:

$$\begin{aligned} B_a(2e_a, 2e_b) &= C_a(e_a, e_b) && (9)\\ B_b(2e_a, 2e_b) &= C_b(e_a, e_b). && (10)\end{aligned}$$

Proposition 1. *When employees are risk neutral, a promotion tournament can induce efficient effort at each task.*

Proof. W_ℓ is determined so that the individual rationality constraint (7) is satisfied in equality. Substituting for W_ℓ, we obtain the problem:

$$\begin{aligned} \max_{x, \Delta W} \qquad & B(2e_a^*, 2e_b^*) - 2(\bar{U} + C(e_a^*, e_b^*))\\ & \text{subject to} \quad (5), (6).\end{aligned}$$

The first-order conditions for this problem are:

$$\begin{aligned} (B_a - C_a)\frac{\partial e_a^*}{\partial x} + (B_b - C_b)\frac{\partial e_b^*}{\partial x} = 0\\ (B_a - C_a)\frac{\partial e_a^*}{\partial \Delta W} + (B_b - C_b)\frac{\partial e_b^*}{\partial \Delta W} = 0.\end{aligned}$$

From Lemma 1, $\frac{\partial e_a^*}{\partial x} > 0$, $\frac{\partial e_b^*}{\partial x} < 0$, $\frac{\partial e_a^*}{\partial \Delta W} > 0$ and $\frac{\partial e_b^*}{\partial \Delta W} > 0$. Thus, we obtain $B_a = C_a$ and $B_b = C_b$. □

The employer is able to choose x to achieve an efficient effort allocation and ΔW to achieve efficient effort levels in both tasks. As in the tournament model with a single task, when employees are risk neutral, there is no efficiency cost from adjusting ΔW to induce efficient effort under optimal x. Although ranking employees is difficult for jobs that include multiple tasks, the multitask incentive problem can be solved by using a promotion rule that involves choosing a winner randomly from among the best performers in each task. This efficient outcome can be achieved for any level of observability of tasks and substitutability in effort costs. The employer does not need to regroup tasks based on observability.

We denote the optimal assessment probability and wage increase as $(x^*, \Delta W^*)$, which satisfies the following conditions:

$$B_a(2e_a^*(x^*, \Delta W^*), 2e_b^*(x^*, \Delta W^*)) = C_a(e_a^*(x^*, \Delta W^*), e_b^*(x^*, \Delta W^*)) \quad (11)$$
$$B_b(2e_a^*(x^*, \Delta W^*), 2e_b^*(x^*, \Delta W^*)) = C_b(e_a^*(x^*, \Delta W^*), e_b^*(x^*, \Delta W^*)). \quad (12)$$

Next, we investigate how the optimal promotion tournament, $(x^*, \Delta W^*)$, varies with the parameter, σ_a.

Proposition 2. *As measuring a task becomes more difficult, the outcome of the task should be increasingly relied upon in the promotion decision, and the wage increase accompanying a promotion should be larger.*

Proof. See Appendix.

When employees engage in multiple tasks, the most important problem is the balance of incentives between tasks. If the incentive for employees to make an effort on one task is stronger than the incentive to do so on another task, then employees tend to devote their efforts to the task that is strongly related to a promotion, or in the worst case, they only do one task. The employer can adjust the promotion rule, x, to balance the incentives between tasks for any uncertainty in our model. Therefore, the employer can achieve an efficient effort allocation, even when the uncertainty of task a is very high, but that of task b is very low. Proposition 2 implies that x should increase as the measurability of task a worsens, that is, as σ_a increases. The increased uncertainty in task a depresses the incentive to exert an effort on task a, and the increase in x reduces the incentives for task b. Therefore, ΔW needs to be larger to compensate for the reduced incentive for both tasks.

In particular, if the employer wishes to achieve the same level of effort on both tasks ($e_a^{FB} = e_b^{FB}$), then x must satisfy:

$$x\frac{\Delta W}{2\sigma_a\sqrt{\pi}} = (1-x)\frac{\Delta W}{2\sigma_b\sqrt{\pi}}$$

from equations (1) and (2). Solving this equation for x, we obtain:

$$x = \frac{\sigma_a}{\sigma_a + \sigma_b} = \frac{1}{1 + \frac{\sigma_b}{\sigma_a}}. \tag{13}$$

Thus, x decreases with σ_b/σ_a when the employer motivates employees to exert the same level of effort for both tasks. Therefore, the promotion rule depends on the relative measurability between both tasks. In the extreme case, where $\sigma_a \approx \infty$ and $\sigma_b \approx 0$, then x approaches 1.

Proposition 2 means that an employee that performs better in the more uncertain task has a higher probability of promotion. Therefore, x can be interpreted as being the intensity of incentive for task a. Proposition 2 implies that the relationship between the production uncertainty and incentive provision is positive. This is the opposite result to that showed by Holmstrom and Milgrom (1987, 1991) in their linear wage model.

On the other hand, some empirical studies have found a positive relationship[4]. This is because there are many factors that affect a compensation scheme in the real world[5]. Marino and Zabojnik (2004) analyzed a tournament between teams, and showed a positive relationship between an explicit incentive and production uncertainty. Our result and that of Marino and Zabojnik (2004) implies that whether or not a wage is based on the absolute performance or a relative compensation scheme is one factor that determines the relationship between the incentive and uncertainty.

4 Extensions

Risk-averse agents The first-best outcome of Proposition 1 depends on the assumption of risk-neutral employees. When employees are risk averse, then the employer faces a trade-off between incentives and insurance. The first-best solution cannot be achieved, because ΔW is constrained by the insurance problem. However, the positive relationship between x and σ_a will be sustained. As the probability of winning the promotion tournament is always $1/2$ for any σ_a at equilibrium, then the expected utility of employees is not decreased, even if the outcome of a higher risk task is used more frequently than the outcome of a lower risk task.

[4]Prendergast (2000) surveyed the empirical relationship between uncertainty and incentives.

[5]For example, Ackerberg and Botticini (2002) examined endogenous matching, where risk-neutral tenants chose risky crops and risk-averse tenants chose lower-risk crops. The risky crops were associated with fixed contracts (a strong incentive) because the agents were risk neutral.

Heterogeneous agents Some tasks are simple enough to be performed by everyone identically, whereas others require either high skill or an ability to complete them. What will happen to our promotion rule if employees are heterogeneous in their ability and the performance of a task depends on their ability? Suppose that the ability of agent i to engage in task a is higher than that of agent j, but that they have the same ability in task b. We use the same model as analyzed in the previous section, except for the outcome of task a. Let us denote the observed measure of employee i in task a as $q_a^i = e_a^i + a^i + \epsilon_a^i$ and that of employee j as $q_a^j = e_a^j + a^j + \epsilon_a^j$, where $a^i > a^j$. $\Delta a \equiv a^i - a^j$ represents the difference in ability. We assume that the employees are aware of each other's ability, but that this information is not available to the employer.

Employee i solves the following problem:

$$Max_{e_a^i, e_b^i} \;\; W_\ell + \Delta W \left\{ x G_a(e_a^i - e_a^j + \Delta a) + (1-x) G_b(e_b^i - e_b^j) \right\} - C(e_a^i, e_b^i).$$

The first-order conditions are:

$$\begin{aligned} x g_a(e_a^i - e_a^j + \Delta a)\Delta W &= C_a^i = e_a^i + \gamma e_b^i && (14) \\ (1-x) g_b(e_b^i - e_b^j)\Delta W &= C_b^i = e_b^i + \gamma e_a^i. && (15) \end{aligned}$$

Employee j solves the following problem:

$$Max_{e_a^j, e_b^j} \;\; W_\ell + \Delta W \left\{ x G_a(e_a^j - e_a^i - \Delta a) + (1-x) G_b(e_b^j - e_b^i) \right\} - C(e_a^j, e_b^j).$$

The first-order conditions are:

$$\begin{aligned} x g_a(e_a^j - e_a^i - \Delta a)\Delta W &= C_a^j = e_a^j + \gamma e_b^j && (16) \\ (1-x) g_b(e_b^j - e_b^i)\Delta W &= C_b^j = e_b^j + \gamma e_a^j. && (17) \end{aligned}$$

Since $g_t(\cdot)(t = a, b)$ is symmetrical at zero, it follows that $e_a^i = e_a^j \equiv \hat{e}_a$ from equations (14) and (16), and that $e_b^i = e_b^j \equiv \hat{e}_b$ from equations (15) and (17). The equilibrium effort in each job is given by:

$$\begin{aligned} \hat{e}_a &= \frac{\Delta W}{(1-\gamma^2)} \left\{ x g_a(\Delta a) - \gamma(1-x) g_b(0) \right\} && (18) \\ \hat{e}_b &= \frac{\Delta W}{(1-\gamma^2)} \left\{ (1-x) g_b(0) - \gamma x g_a(\Delta a) \right\}. && (19) \end{aligned}$$

$g_t(\cdot)$ has maximal value at $\Delta a = 0$. Thus, from conditions (18) and (19), we obtain the next proposition.

Proposition 3. *Compared to the case where skills are identical, the level of effort devoted to the task dependent on an individual's ability decreases, and the level of effort devoted to the task that is not dependent on an individual's ability increases.*

Lazear and Rosen (1981) showed that a mixed tournament is less efficient than a homogeneous tournament because it reduces the incentives of the less capable employee and also the incentives of the capable employee. The same reasoning is applied to task a. Employee i has an advantage in task a, and they can perform it better and more easily than can employee j can. Employee i does not need to devote much effort to win at task a, whereas employee j tends to give up, knowing that he or she is unlikely to win at task a. Therefore, the incentive for performing task a decreases. However, there is an additional effect in this multitask model. As the two tasks are substitutes in the cost function, the decreased effort for task a induces increased effort for task b. Therefore, the employees tend to devote greater effort to the task in which they are equally matched.

The individual rationality constraint of employee i is not binding and the more able employee obtains some rent, because employee i has a higher probability of winning the promotion tournament than employee j has for any x. Therefore, the first-best solution cannot be implemented.

We will consider the promotion rule, x, that makes employees allocate the same level of effort to both tasks. Then, conditions (18) and (19) imply that x must be set to satisfy the following equation:

$$x = \frac{g_b(0)}{g_a(\Delta a) + g_b(0)}. \tag{20}$$

Note that $g_a(\Delta a)$ decreases with Δa. We then have the following proposition.

Proposition 4. *The outcome of a task in which performance depends more heavily on the contestants' ability should be relied upon more in making the promotion decision.*

If the employees' abilities are more heterogeneous in performing one task than another task, then Proposition 4 implies that the motivations for employees to make efforts on the task that depend on an employees' ability are weaker. To defuse this incentive problem, the promotion decision should be based more heavily on the information regarding this task. This result is consistent with intuition. The firm should attach greater importance to the performance of an ability-demanding task in the promotion decision than to the performance of an easy task.

5 Conclusions

In this paper, we have investigated a promotion tournament with multiple tasks. First, we analyzed the case of risk-neutral homogeneous agents. In this case, the employees could achieve the first-best outcome by randomly choosing the winner of the promotion from among the best performers in each task. The probability that the best performer in a task becomes the winner depended on the observability of the task. We showed that when measuring the performance of the task is more difficult, the promotion decision should be based more heavily on the outcome of a match regarding the task. In Section 4, we analyzed the case where employees are heterogeneous in terms of their ability to undertake one of the tasks. We showed that when the difference in ability was large, then the outcome of a match in the ability-dependent task should be relied upon more heavily in making the promotion decision.

In a wage contract based on an absolute output, designing the job is important because grouping tasks seriously affects an employee's incentive. On the other hand, our promotion rule is an effective device to provide employees with incentives for all jobs in which tasks are heterogeneous with respect to the ease of measuring performance. The employer can organize the tasks into jobs from the viewpoint of production efficiency. Therefore, our analysis proposes one reason why using promotion to motivate employees is better than using an output-based wage scheme.

Appendix

Proof of Proposition 2 From (11) and (12), the optimal promotion tournament $(x^*, \Delta W^*)$ satisfies following conditions:

$$F_a \equiv B_a(2e_a^*(x^*, \Delta W^*), 2e_b^*(x^*, \Delta W^*)) - e_a^*(x^*, \Delta W^*) - \gamma e_b^*(x^*, \Delta W^*) = 0$$
$$F_b \equiv B_b(2e_a^*(x^*, \Delta W^*), 2e_b^*(x^*, \Delta W^*)) - e_b^*(x^*, \Delta W^*) - \gamma e_a^*(x^*, \Delta W^*)) = 0.$$

Differentiating F_a and F_b with respect to σ_a and applying the implicit function theorem, we obtain

$$\frac{\partial x^*}{\partial \sigma_a} = \frac{1}{|J|}\left\{-\frac{\partial F_a}{\partial \sigma_a}\frac{\partial F_b}{\partial \Delta W} + \frac{\partial F_a}{\partial \Delta W}\frac{\partial F_b}{\partial \sigma_a}\right\} \tag{A-1}$$

$$\frac{\partial \Delta W^*}{\partial \sigma_a} = \frac{1}{|J|}\left\{-\frac{\partial F_a}{\partial x}\frac{\partial F_b}{\partial \sigma_a} + \frac{\partial F_a}{\partial \sigma_a}\frac{\partial F_b}{\partial x}\right\}. \tag{A-2}$$

The Jacobian determinant can be calculated as

$$|J| = 2(B_{aa}B_{bb} - Bab^2) + 1 - \gamma^2 + 2(-B_{aa} - B_{bb} + 2\gamma B_{ab}).$$

From $B(\cdot)$ is concave function, $B_{ab} > 0$ and $\gamma < 1$, it follows that $\mid J \mid > 0$. The term in braces in (A-1) is calculated as

$$
\begin{aligned}
&-\frac{\partial F_a}{\partial \sigma_a}\frac{\partial F_b}{\partial \Delta W} + \frac{\partial F_a}{\partial \Delta W}\frac{\partial F_b}{\partial \sigma_a} \\
&\quad = \left\{ -\frac{\partial e_a^*}{\partial \sigma_a}\frac{\partial e_b^*}{\partial \Delta W} + \frac{\partial e_b^*}{\partial \sigma_a}\frac{\partial e_a^*}{\partial \Delta W} \right\} \left\{ (2Baa-1)(2B_{bb}-1) - (2B_{ab}-\gamma)^2 \right\} > 0.
\end{aligned}
$$

Hence, we have $\frac{\partial x^*}{\partial \sigma_a} > 0$.

The term in braces in (A-2) is calculated as

$$
\begin{aligned}
&-\frac{\partial F_a}{\partial x}\frac{\partial F_b}{\partial \sigma_a} + \frac{\partial F_a}{\partial \sigma_a}\frac{\partial F_b}{\partial x} \\
&\quad = \left\{ -\frac{\partial e_a^*}{\partial x}\frac{\partial e_b^*}{\partial \sigma_a} + \frac{\partial e_a^*}{\partial \sigma_a}\frac{\partial e_b^*}{\partial x} \right\} \left\{ (2Baa-1)(2B_{bb}-1) - (2B_{ab}-\gamma)^2 \right\} \\
&\quad = \frac{\partial e_a^*}{\partial \sigma_a}\frac{\Delta W}{(1-\gamma^2)\sqrt{2\pi}}\frac{\gamma^2-1}{\sigma_b} > 0.
\end{aligned}
$$

It follows that $\frac{\partial \Delta W^*}{\partial \sigma_a} > 0$.

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