Birkbeck Working Papers in Economics & Finance

ISSN 1745-8587

Birkbeck
UNIVERSITY OF LONDON

School of Economics, Mathematics and Statistics

BWPEF 0915

An essay on the generational effect of employment protection

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November 2009

Revised version, 24 November 2009

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Abstract

This paper provides an explanation for the observed positive relationship between youth unemployment and the cost of firing workers. When the cost of firing workers is high, firms only fire when the present discounted value of future losses is high, in which case they gain little by postponing the firing decision in the hope that productivity will recover. The young workers are then the first to go due to their longer remaining tenure. In contrast, when the cost of firing workers is low, the present discounted value of future losses is small at the firing margin and firms may choose to wait in the hope of a recovery. In this case they may choose to fire the older workers first since the younger ones are more likely to be around when productivity recovers.

Keywords: Age-structure, tenure, firing decisions, real options.
JEL: E32, J23, J24, J54

We are grateful to two anonymous referees for very useful comments on an earlier draft of this paper.

Firms have the option of choosing when to fire workers during economic downturns. This option has implications for the composition of the pool of workers fired. In particular, we will show how the interplay of uncertainty about future productivity and the level of firing costs affects the age composition of the pool of fired workers. Our intuition is simple: in a perfect-foresight framework, management may decide to fire its younger workers in a downturn if it does not expect profits to recover – because the present discounted value of future losses from employing them exceeds that for the older workers due to longer expected tenure – while with uncertainty about future productivity and profits, management may decide to wait before firing the young workers, the more so the greater is the uncertainty. We call the first case the "tenure effect" and the second the "sacrificed options effect". We will show how the level of firing costs determines their relative size; in particular how high firing costs can make the tenure effect dominate the sacrificed options effect and firms respond to a downturn by firing the younger workers.

The decision by firms to fire workers is essentially an irreversible investment decision under uncertainty, whose timing can be chosen by management. Severance payments to labour are one obvious reason why the firing decision is most accurately modelled as an investment decision. There are also implicit costs of firing workers in the absence of such laws. For example, firing decisions may disrupt production, have negative morale effects or shorten the expected future tenure of remaining workers. A simple way of modelling the firing decision is to compare the present discounted value of future losses from employing a worker with the cost of firing him – a Tobin-q theory of dismissals. This insight gives a *tenure effect* so that firms gain more – in terms of lowering expected discounted losses – from firing young workers with long remaining tenure. But it has been observed that firms only fire workers when the expected savings from doing so far exceed the cost, which indicates that they take into account the possibility that profits may recover and that they will live to regret the firing decision. This observation is captured by Okun's law. Our intuition is simple: with low firing costs the option to fire a worker is valueable and the firm may hesitate to fire a given worker because productivity and profits may improve in the future. The effect will be to protect the employment of young workers – who have long expected job tenure – at the expense of older workers. Thus the *sacrificed option effect* is greater for the younger workers because it is more likely that productivity will recover during their tenure. In contrast, older workers may be fired because it is less likely that a recovery may lead the firm management to regret such a decision in light of their short remaining tenure. However, when the level of the firing costs goes up this option becomes less valuable and the employment of the younger workers is threatened. In sum, while the tenure effect dominates at high firing costs the sacrificed option effect dominates at low firing costs.

Intuitively, at the firing margin, the present discounted value of future losses from an employed worker is greater the higher are firing costs, making the firm prefer to fire workers with longer expected tenure. The tenure effect dominates the sacrificed option effect. But when the firing costs are low, the expected discounted losses are small at the firing margin, and a small recovery of productivity may turn losses into profits and the sacrificed option effect becomes stronger. This is more likely to happen in the case of a young worker.

When considering the hiring decision, in spite of the marginal benefit of hiring young workers falling with the level of firing costs, firms prefer to hire the younger workers first for all vales of firing costs because they have longer expected tenure and also, and for the same reason, a larger firing option than the older workers. Therefore, our analysis has the empirical implication of a positive relationship between the relative unemployment of young workers and the level of firing costs when wages are treated as exogenous.[1]

Our model complements that of Lazear and Freeman (1997) who find that during a downturn the youngest and the oldest workers should be the first to go because the young have not been given any firm-specific skills while the productivity of older workers has declined relative to their wages. While this can explain high youth unemployment during recessions, it does not predict a positive relationship between the level of firing costs and youth unemployment.

A number of studies have attempted to estimate the extent to which the poor performance of European countries can be explained by formal employment-protection legislation. Lazear (1990) uses a sample of 22 OECD countries over the period 1956 and 1984 and finds that severance-pay requirements reduce employment.[2] Our model has the empirical prediction of a positive relationship between relative youth unemployment and the level of firing costs. These empirical predictions are supported by a number of papers. While Blanchard (2006) emphasises the magnitude of the youth unemployment problem in Europe, Bertola et al. (2002) find that high unemployment for the younger age groups is particularly pronounced in the more unionised countries as well as in those having more stringent employment protection legislation. Such a relationship has also been found by Scarpetta (1996) and Jimeno and Rodriguez- Palenzuela (2003). Scarpetta (1996) estimates

[1] In a model with endogenous wages, there is of course the possibility of wages adjusting to ensure indifference. However, it is clear that firms are not indifferent as to the age composition of the staff they choose to fire. Clearly, there are other effects on wage setting; such as the promise of rising wage profiles intended to spur effort, which has been used to explain mandatory retirement (Lazear, 1979), as well as firms using tournaments to increase effort (Lazear and Rosen, 1981) or unions having preferences about the relative wages of different age groups.

[2] See also Scarpetta (1996), Elmeskov, Martin and Scarpetta (1998), Nickell (1998), DiTella and MacCulloch (1998) and Nickell, Nunziata and Ochel (2005).

unemployment equations for OECD countries for the period 1983-1993 for both the total unemployment rate as well as the youth unemployment rate, the long-term unemployment rate and the non-employment rate. He finds support for the earlier result of Lazear that stringent employment protection contributes to high unemployment and non-employment rates. Moreover, his results suggest that employment protection raises youth and long-term unemployment. In addition, young workers are adversely affected by strong unions who may set wages above their market-clearing level. Jimeno and Rodriguez-Palenzuela (2003) use a panel data set for OECD countries in the period 1960 to 1996 to measure the effect of macroeconomic shocks, labour market institutions and demographic development to explain the relative youth unemployment rate. The macroeconomic factors considered include labour demand shifts, real interest rates and total factor productivity growth while labour market institutions include unemployment benefit systems, a measure of active labour market policies, wage determination, the tax wedge, employment protection, minimum wages and an indicator of the strictness of the legislation regarding the use of temporary contracts. The results show that the youth unemployment differential (defined as the difference in the rate of unemployment between the 15-25 age group and the 25-54 age group) increases with the strictness of employment protection and the strictness of regulation affecting temporary employment – meaning the ease by which workers get fixed-term contracts – in addition to union density, the tax wedge and decreases in coordination. Thus the youth unemployment differential is again shown to be an increasing function of the strictness of the employment protection legislation.

Our model belongs to a body of research that uses real options to describe the effect of different macroeconomic variables, such as real interest rates and expected productivity growth rates, on the firing decision (see Bentolila and Bertola, 1990).[3] Our contribution is to model the firing decision by allowing for worker heterogeneity in terms of age.

1. An option-valuation approach

The option-valuation approach to investment has been popular since the seminal papers of Black and Scholes (1973) and Merton (1973) on the pricing of stock options. These methods of valuing stocks can be easily applied to *real options*, which denote the option-like characteristics of investment opportunities. The decision to invest (or the decision to exercise real options) becomes important with the existence of uncertainty and sunk costs. McDonald and Siegel (1986) show that the required rate of return on investment in many large industrial projects can be more than doubled by moderate amounts of uncertainty when the investment project is at least partly irreversible.[4]

In most cases it is assumed that the real options are infinitely lived – the real-life investment opportunities are infinitely lived and never valueless (e.g. McDonald and Siegel; 1984, 1986). However, some research deals with the non-perpetual real options (e.g. Paddock, Siegel and Smith, 1988). However, it has been claimed that it is often not possible to solve such non-perpetual options analytically, making numerical methods essential.[5] Generally speaking, it is hard to solve for free-

[3] In another paper (Chen and Zoega, 1999) we have shown how the employment effects of employment protection depend on the nature of the stochastic process followed by productivity – trend growth, variance, and degree of mean reversion – in addition to the real rate of interest and workers' quit rates.
[4] For an introduction, see Dixit and Pindyck (1994).
[5] One of the reasons for the non-existence of analytical solutions is that such options are similar to *American* stock options that can be exercised at any time up to the expiration date. It is well-known that American stock options can only be solved for using analytical approximations or numerical methods such as finite-difference methods. American call options with lump-sum dividends are an exception though in that their terminal and boundary conditions differ (see Roll, 1977; Geske, 1979;

boundary time-dependent real options. This is partly due to functions of time-dependent options having a complicated shape, which may require several analytical functions for simulation. We will show in the case of real options that *approximate* analytical solutions do exist.[6] The approximate solutions of non-perpetual real options should share the same composite components as perpetual real options. The partial differential equation of non-perpetual real options can then be transformed into a convection-diffusion problem,[7] which can be solved for analytically using partial differential equations.

2. Modelling the firing decision

There is only one sector in our economy that uses labour as an input to produce a homogenous good. Since our focus is on labour demand, real wages are assumed to be fixed and their determination is not described. The source of uncertainty is stochastic productivity. Current profits, measured in units of output, are defined as follows,

$$\Pi(g_t, N_t) = g_t N_t^{\theta} - wN_t, \qquad 0<\theta<1, \tag{1}$$

where N denotes the number of employed workers, w is the real wage, and g is a measure of productivity. It is assumed that each worker has a maximum working life of T at time zero (t=0). Some workers will however end up working less time because they either choose to quit or are fired before reaching this age. To simplify the model, we assume that all workers have the same productivity and wage independent of their

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Whaley, 1981). The possible analytical solutions to partial differential equations vary greatly when boundary- and terminal (and/or initial) conditions change. Changes in such conditions can result in the non-existence of analytical solutions. The method used here is most similar to Barone-Adesi and Whaley (1987).

[6] '*Approximate*' in the sense that the solutions might not be complete but still a good proxy for real solutions. We show in Appendix D the difference between the approximiate solutions and numerical solutions by the finite difference method.

[7] In physics, *convection* is the movement of the substance by the movement of the medium. Combined with a diffusion problem, it will be like the diffusion of a moving wave.

age. Later we make wages depend on age in order to take into account the possibility that workers may become more expensive as they age, as in Lazear (1979).

It is assumed that g follows a geometric Brownian motion

$$dg_s = \eta g_s ds + \sigma g_s dW_s \,, \tag{2}$$

where W_s is a standard Wiener process; $dW_s = \varepsilon_s \sqrt{ds}$ and ε_s is a serially uncorrelated, normally distributed random variable with mean zero and a standard deviation of unity. Here η is the drift parameter (the expected growth rate of labour productivity) and σ the variance parameter. We initially assume that this average quit rate per unit time is constant over time and equal to λ and later make it age dependent. The probability that a given worker will quit over the interval ds is therefore equal to λds.

The firm's expected marginal value of an employee without any firing and/or hiring is

$$v(Y,t;T) \equiv v(Y_t,t;T) = E\left[\int_t^T (Y_s - w)e^{-(\rho+\lambda)(s-t)}ds\right], \tag{3}$$

where $E[\cdot]$ denotes the expectation operator given the information set available to the firm at time t, ρ denotes the real interest rate, v is the (intertemporal) marginal value of workers, $Y_s = \theta g_s N_s^{\theta-1}$ represents the marginal product of labour at time s, and $Y_s - w$ denotes instantaneous marginal profits at time s. Equation (3) is similar to the expressions in Bentolila and Bertola (1990) except each marginal worker can only work for the firm for a maximum interval of $(T - t)$. Note that $(T - t)$ is the maximum possible tenure since workers might quit or get fired earlier. Itô's Lemma gives the following process for marginal productivity,

$$dY = \eta_Y Y ds + \sigma Y dW \,, \tag{4}$$

where $\eta_Y = \eta + \lambda(1-\theta)$.

Now consider the effect of the firing and hiring costs on firms' profits decisions. When the expected marginal value of an employee is greater than the hiring cost, the firm starts to hire new workers; when the negative of the expected marginal value of an employee is higher than the firing cost, the firm starts to fire workers. Thus, the process of Y_s or $v(Y,t)$ becomes an *optional stopping problem* or *regulated Itô process*. The firm will hire a marginal worker if

$$v(Y,t;T) \geq H \tag{5.1}$$

and fire a marginal worker if

$$-v(Y,t;T) \geq F\,, \tag{5.2}$$

where *H* and *F* represent hiring and firing costs respectively.

A standard technique for solving the above dynamic optimisation problem is Bellman's Principle of Optimality (Bellman, 1957). Using Itô's Lemma, we get the following Bellman equation for the marginal value of the firm's stock of workers; $v \equiv v(Y,t;T)$, in the continuation region where the values of future hires and fires are not taken into account,

$$(\rho+\lambda)v = Y - w + \eta_Y Y v_Y + \frac{1}{2}\sigma^2 Y^2 v_{YY} + v_t\,. \tag{6}$$

Equation (6) relates the value of the marginal workers to the value of the stochastic variable *Y* at each point in time. The partial term with respect to time, v_t, in equation (6) makes the differential equation difficult to solve. A simple way to get around this without resorting to pure numerical methods would be to assume that the analytical solutions have the same components as the infinitely-lived case found in Bentolila and Bertola (1990). In this case, equation (6) becomes a second-order ordinary differential equation in Y and as a result the option values of hiring and firing workers become independent of time. It follows that the options for hires and/or fires do not approach zero when workers age. In this paper we will correct for this and show how interesting implications arise.

The problem now is to solve for v, which is the value of employing a marginal worker. The solution for v consists of the particular integral and the general function. A convenient particular solution, v^P, for (6) can be obtained by integrating (3) directly

$$v^P = aY - bw, \tag{7}$$

where $a = \left(1 - e^{-(\rho+\lambda-\eta_Y)(T-t)}\right) / (\rho + \lambda - \eta_Y)$, $b = \left(1 - e^{-(\rho+\lambda)(T-t)}\right) / (\rho + \lambda)$ and it is assumed that the denominator of the parameter a is positive. As T approaches infinity, the particular solution becomes identical to the one in the perpetual setup. The smaller value of T yields smaller value of particular solutions, which echoes the intuition that old workers have a lower value for the firm.

The firm takes into account the option value of hiring in the future. There is also the option to fire the worker once he is employed. The two option values are measured by the general (or homogenous) solutions to equation (6). Now only focusing on the homogenous part of (6) and letting v^G be the value of the marginal option, we get

$$(\rho + \lambda) v^G = \eta_Y Y v_Y^G + \frac{1}{2}\sigma^2 Y^2 v_{YY}^G + v_t^G . \tag{8}$$

The general solutions of (8) are equal to the value of the options to hire or fire the marginal worker. When Y approaches zero, the value of the option to hire, v_H^G, has to go to zero. Similarly, the firing option, v_F^G, is equal to zero when Y goes to infinity. Thus, the general solutions for the hiring and firing options have to satisfy the following boundary conditions respectively,

$$\lim_{Y\to 0} v^G(Y,t;T) = 0 \text{ for the hiring option,} \tag{9.1}$$

$$\lim_{Y\to \infty} v^G(Y,t;T) = 0 \text{ for the firing option.} \tag{9.2}$$

A special case of equation (8) is when workers live forever (T=∞). Thus, the term v_t^G in equation (8) disappears and the values of the hiring- and firing options are (see Appendix A)

$$v_H = A_1 Y^{\beta_1} \text{ for hiring option,} \tag{10.1}$$

$$v_F = A_2 Y^{\beta_2} \text{ for firing option.} \tag{10.2}$$

The unknown parameters of A_1 and A_2 are determined by the value-matching and smooth-pasting conditions and β is determined by equation (11).

$$\frac{1}{2}\sigma^2 \beta(\beta - 1) + \eta_Y \beta - (\rho + \lambda) = 0\,, \tag{11}$$

where β_1 and β_2 are positive and negative roots of the above equation respectively. The general solutions to (8) are then given by the following equations (see Appendix B):

$$v_H^G(Y,t;T) = A_1 Y^{\beta_1} N(d_1)\,, \tag{12.1}$$

$$v_F^G(Y,t;T) = A_2 Y^{\beta_2} N(-d_2)\,, \tag{12.2}$$

where A_1 , A_2 are unknown parameters, $d_{1/2} = \dfrac{\ln Y \pm \sigma^2 (T-t)\sqrt{\left(\dfrac{\eta_Y}{\sigma^2} - \dfrac{1}{2}\right)^2 + \dfrac{2(\rho+\lambda)}{\sigma^2}}}{\sigma\sqrt{T-t}}$,

and $N(d) = \left(1/\sqrt{2\pi}\right)\int_{-\infty}^{d} e^{-\varpi^2/2} d\varpi,\ 0 \le N(d) \le 1$, is the cumulative normal distribution function. The general solutions, or the real options to hire/fire, have the components similar to the ones of financial options such as in Black and Scholes (1973) while keeping its perpetual parts components. Such functional forms of general solutions make it possible to use the general solutions to solve the optimal stopping problem of hiring and firing.

Looking at the hiring- and firing options we find two separate cases:

Case 1: $T \to \infty$

It is easy to show that as T approaches infinity (workers live forever), the cumulative distribution functions of $N(d_1)$ and $N(-d_2)$ become unity. This reduces the firing- and hiring options to the case of perpetual options.

Case 2: $T \to t$

If $\ln Y$ >0, then $N(d_1)$=1 and $N(-d_2)$=0 as T approaches t. If the marginal profitability is high enough, firms mainly focus on the hiring decision. The firing option approaches zero because these workers will retire very soon.

If $\ln Y$ <0, then $N(d_1)$=0 and $N(-d_2)$=1 as T approaches zero. A small $(T-t)$ means that Y needs to be very small to reach the firing threshold. If the marginal profitability is low, firms mainly consider the firing decision.

The decision as to hire or fire workers depends on his value as given by the equations giving particular solutions and homogenous solutions: (7), (12.1), and (12.2). The definition of the firing- and hiring barriers; Y_F and Y_H, are then given by the value-matching and smooth-pasting conditions:

Value-matching conditions

$$aY_H - bw + v_F^G\left(Y_H, t; T, A_2\right) = H + v_H^G\left(Y_H, t; T, A_1\right), \tag{13}$$

$$-\left(aY_F - bw\right) + v_H^G\left(Y_F, t; T, A_1\right) = F + v_F^G\left(Y_F, t; T, A_2\right). \tag{14}$$

The left-hand side of (13) has the marginal benefit of hiring which includes the acquired firing option. The right-hand side has the marginal cost of hiring, which includes the sacrificed hiring option. Similarly for equation (14), the left-hand side has the marginal benefit and the right-hand side the marginal cost of firing. In our numerical solutions below – apart from Figure 1 of the general case of hiring and firing – we will only include the sacrificed firing option as part of the cost of firing; we will not include the acquired hiring option as a benefit of firing. In this case, we do not consider the hiring decision so that once an employee gets fired, the firm never hires this employee back – the value of the hiring option is set equal to zero. The reason for doing this is that firing workers is not going to alter a firm's chances of filling a vacancy in the future if there are many (homogeneous) unemployed people (of each age) to start with.

There are four unknown variables, Y_H, Y_F, A_1, and A_2, in equations (13) and (14). The smooth-pasting conditions follow to ensure the slopes before and after thresholds with respect to Y_H and Y_F are the same:

Smooth-pasting conditions

$$a+\frac{\partial v_F^G\left(Y_H,t;T,A_2\right)}{\partial Y_H}-\frac{\partial v_H^G\left(Y_H,t;T,A_1\right)}{\partial Y_H}=0\,, \tag{15}$$

$$a+\frac{\partial v_F^G\left(Y_F,t;T,A_2\right)}{\partial Y_F}-\frac{\partial v_H^G\left(Y_F,t;T,A_1\right)}{\partial Y_F}=0\,, \tag{16}$$

where details of derivations of various $\partial v^G \big/ \partial Y$ are shown in Appendix C.

Equations (13), (14), (15) and (16) are a non-linear systematic equations with four unknown parameters [Y_H, Y_F, A_1, and A_2] and can be solved for numerically, once beta roots, β_1 and β_2, are obtained from equation (11).

3. Hiring and firing

We will now calculate the firing thresholds on the basis of equations (13), (14), (15) and (16). We calculate the hiring and the firing thresholds for a fixed level of firing costs in the two-threshold case when both the hiring- and the firing thresholds are calculated simultaneously. We can show the effect of firing costs on hiring and firing by plotting firing thresholds against firing costs. This is shown in Figure 1 below.[8] We assume that workers enter the labour market at age 20 and retire at age 65 – it follows that a worker with *T-t* =25, that is someone with 25 years of working life remaining, is currently 40 years of age.

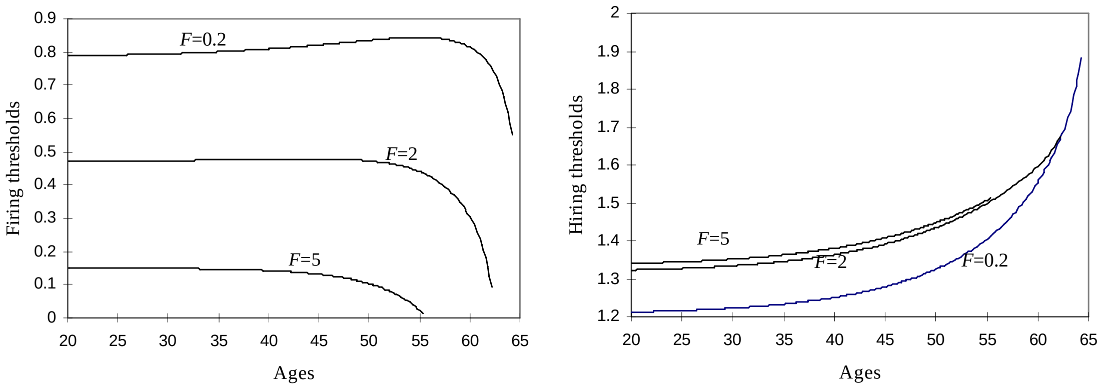

Figure 1.The effect of age on the hiring- and firing thresholds with different effective firing costs. Age is equal to (65–*T*). Other parameters: σ=0.20, ρ=0.10, θ=0.7, η=0.02, λ=0.05, *w*=1, *H* = 0.083, and *t*=0.

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[8] In Appendix D, we discuss the difference between the approximate solutions and pure numerical solutions by the standard explicit finite difference method. The results, if anything, show that the real thresholds of firing are greater than the ones of the approximate solutions for the old. This shows that the results of the paper hold.

As the effective firing costs rise, the firm becomes more inclined to fire the younger among its workers. The reason is that part of the cost of firing workers is the sacrificed option of doing so in the future. This was shown in equation (14). This firing option is decreasing in both the level of the firing costs and in the worker's age. For low levels of firing costs, the marginal cost of firing the young workers is higher than the cost of firing older ones for this reason. But at high firing costs, the difference is much smaller as the firing option is always very low – both for the young and the old workers. However, the marginal benefit of firing the young workers is always higher – that is for all levels of firing costs – because of their longer remaining tenure. We conclude that the firm would choose to fire the young workers first if firing costs were high – the value of the firing option low – but at low firing costs it may choose to fire the older workers first since the marginal cost of doing so is much lower. Furthermore, we find that firms always hire younger workers first no matter what level the firing costs are.

We can explain the effect of *F* on firing options in more details in the following simplified case. Consider the firing-only scenario with perpetual workers, the value-matching and smooth conditions are as follows

$$-\left(aY_F - bw\right) = F + A_2 Y_F^{\beta_2}, \tag{17}$$

$$a = \beta_2 A_2 Y_F^{\beta_2 - 1}, \tag{18}$$

where $A_2 Y_F^{\beta_2}$ represents the perpetual firing option as shown in (10.2). Equation (18) shows that the value of the option to fire is a linear function of *Y*: $aY/\beta_2 = A_2 Y_F^{\beta_2}$. Thus, if the firing threshold *Y* falls due to a direct increase of *F*, the option to fire would also decrease accordingly. Though the fall of the *Y* thresholds due to an increase in *F* leads to higher value of $Y_F^{\beta_2}$, the parameter A_2 ensures that the whole firing option falls as F increases. The firing option measures the waiting values from postponing exercising the option to fire workers. As the direct costs of firing F increase, the probability that Y will return to a profitable value for the firm after the firing decision is made becomes smaller. Intuitively, at the firing margin, the present discounted value of future profits from an employed worker is more negative the higher are firing costs, making the firm prefer to fire workers with longer expected tenure. The tenure effect dominates the sacrificed option effect. But when the firing costs are low, the expected discounted losses are small at the firing margin, and a small recovery of productivity may turn losses into profits, the sacrificed option effect becomes stronger. This is more likely to happen in the case of a young worker. Therefore, for a high value of F, the firm would not regret its decisions to fire, which implies that the value of waiting, that is the firing option, is less important to the firm. This relationship still holds numerically for workers who do not live forever and for the system of the general hiring/firing value-matching conditions of equations (13) and (14).

In this sense, the difference between the option of firing a young and an old worker becomes negligible for high F. This implies that with high firing costs, the firm tends to fire the young workers first in order to reduce its losses for a much longer period of time than when the older workers are fired. However with low firing costs, the firm values the option to be able to fire the workers at a later date when more information about the evolution of productivity is available. This option is worth more in the case of the young workers and hence the firm faces higher costs of firing the younger workers on this account. As a consequence, the firm tends to fire the older ones first when the cost of firing is low.

Note the difference between our setup and that of Lazear and Freeman (1997). They claim that it is optimal to fire the younger workers because they are less productive since the (firm-specific) skill accumulation has not been completed. We find that they should also – if there are significant costs of firing – be the first to go even if their productivity is no lower than that of older workers. Firing a young worker is more profitable than firing an older worker since his/her expected tenure is longer. Note also, that these results do not depend on firing costs rising over tenure. All that is needed is a high and constant level of firing costs.

4. Macroeconomic implications

We have found that high firing costs provide more protection to the older workers than to the younger ones. It follows that the age structure of the population affects the tightness of employment-protection legislation; the ageing of the workforce has the same effect on the firing thresholds as an increase in the firing costs themselves. This has two implications.

First, when assessing the nature of a country's labour-market institutions one has to normalise for the age structure of the labour force. Two countries with similar legislation can nevertheless have different effective legislation in the sense that firms are more reluctant to fire workers in one of the countries. Second, changes in the age structure of the population over time may have important consequences. To take an example, the employment-protection legislation already in place in France, Italy and Spain may have been less restrictive in the 1960s and 1970s than in the 1980s and 1990s due to the aging of the baby boom generation.

Figure 2 shows the interaction between the level of firing costs, on the one hand, and the age of workers, on the other hand, in determining the level of productivity at which firms start firing each worker. With low firing costs, the firing threshold is monotonically rising in age making the more mature workers be the first to lose their jobs in a downturn. But as the level of firing costs rises and, the sign of this relationship changes and the threshold becomes monotonically falling in age making the young workers the first to go if labour demand falls.[9]

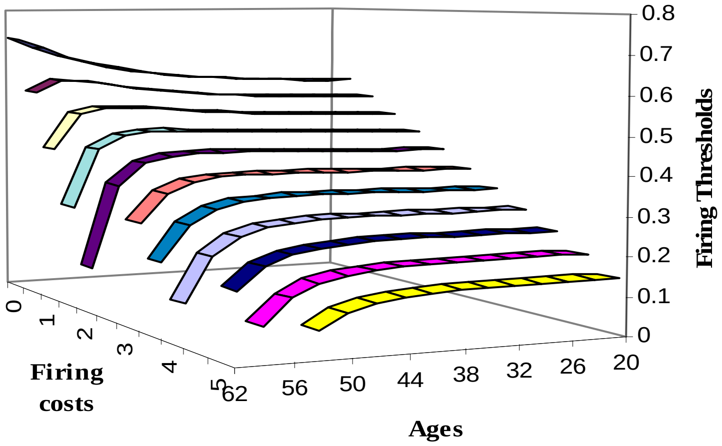

Figure 2.The effect of age on the firing threshold with different firing costs. Age is equal to (65–T). Other parameters: σ=0.20, ρ=0.10, θ=0.7, η=0.02, λ=0.05, w=1, and t=0.

Figure 3 below shows the firing thresholds for both young (age 20) and old (age 60) workers who both will retire at age 65. The figure shows clearly that we fire old workers first with low F and then tend to fire the younger ones as F becomes high, as discussed in the previous section. The thresholds cross where F roughly equals one month's wages, that is F=1.

[9] Note that the numerical results in Figures 2, 3, and 4 are obtained by running the value-matching/smooth-pasting conditions for *firing* thresholds only, with the assumption of null hiring options and no hiring decisions. As shown in Chen and Zoega (1999), this simplification does not affect the qualitative results for firing thresholds when only discussing the effect of F.

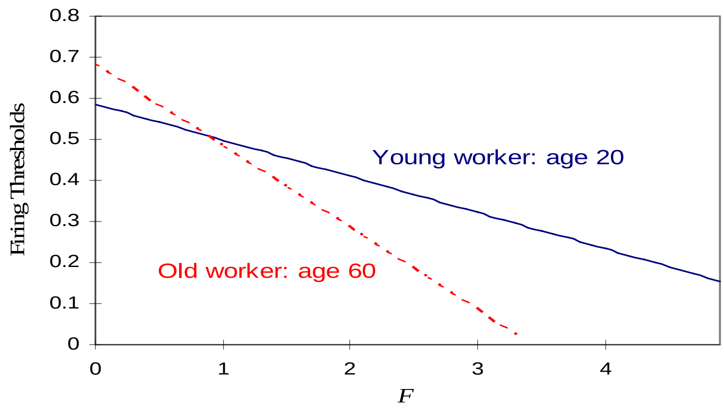

Figure 3.The effect of firing costs on the firing threshold for two age groups. Other parameters: σ=0.20, ρ=0.10, θ=0.7, η=0.02, λ=0.05, w=1, and t=0.

5. Pensions and quits

Other reasons for age discrimination in firing involve the direct and indirect effects of pension schemes. Most pension schemes are defined-benefit schemes, making benefits increase more rapidly as the age of retirement approaches. This makes employers want to lay off workers with a long tenure.[10] However, an extension of our model also predicts that firms may not fire the older workers but for a different reason: When the age of retirement approaches, the older workers have more to gain from quitting and as their quit rates rise, firms benefit more in present discounted value terms from firing the younger workers.

We now augment the model by taking into account the observation that the old are more likely to quit because of the higher pension benefits they can expect to collect in retirement. In particular, we assume that benefits increase more rapidly as the age of retirement approaches. This is described by the following quadratic function:

$$\lambda = 0.05 + 0.0001(45 - (T - t))^2 \qquad (19)$$

This implies that a person of age 20 has a quit rate 0.05, and a person of age 65 has a quite rate 0.22. The effect of introducing this quit rate function[11] is shown in the following diagram:

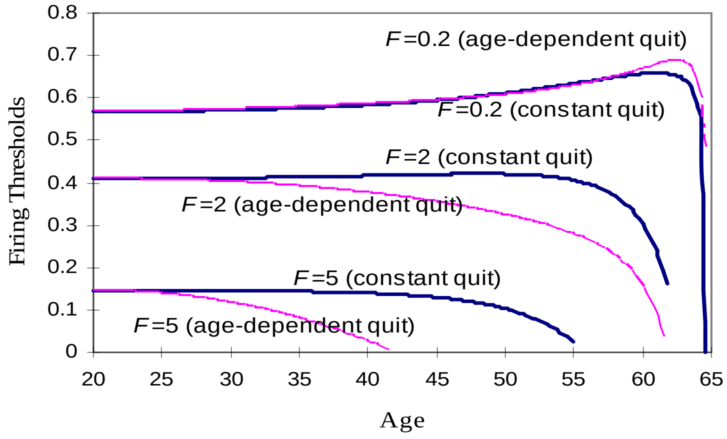

Figure 4.The effect of age on the firing thresholds with different effective firing costs (firing-only scenario). Ages are equal to (65–T). Other parameters: σ=0.20, ρ=0.10, θ=0.7, η=0.02, w=1, H = 0.083, t=0 and λ=0.05+0.0001(45-(T-t))^2. The bolder lines show the case of constant quit rates.

There are two forces at work. As workers become older the present discounted value of future profits (losses) from employing them will fall not just because they have less time left but because they are more likely to quit. This shows up in the value of the particular integral falling with age. The effect is to lower the firing threshold, productivity has to fall to an even lower level than before in the case of the older workers for the firm to want to fire them. Another effect is to lower the value of the firing options, which results in a higher firing threshold. For low levels of firing costs, for example F =0.2, those two effects almost cancel each other out. For higher firing

[10] A possible offsetting effect can be found in Orszag et al. (1999) who show that old workers may behave differently, in particular exert more effort because they have more to lose in the event of a dismissal.

[11] We only consider a firing threshold setup only here. The omittance of hiring thresholds means that there are no hiring option when it comes to the firing decisions. As hiring options have a smaller indirect effect on firing thresholds than fixed hiring costs, compared with the greater value of firing options, the firing threshold only setup will display similar qualitative results except for the downward

costs, the importance of the firing options fades away as discussed in the previous sections. Therefore, the effect on the particular integrals dominates and leads to lower firing thresholds. Thus, the effect of a pension scheme on quitting will reinforce the preference for firing the young workers when the cost of firing is high: Pensions for the old cause job insecurity for the young. Intuitively, firing the young increases the value of the firm by more than firing the old – it is a better investment – because it is less likely that they would have left the firm on their own accord.

6. Rising wages and firing costs

In this section we consider the implications of having wages and firing costs rise with age. Rising wages have been used to justify firms choosing to fire older workers during downturns (Lazear and Freeman, 1997) and also mandatory retirement (see Lazear, 1979), the idea being that while a rising age profile induces workers to stay with their current employer and avoid shirking, there comes a time at which the older workers have become expensive for the firm – wages high relative to productivity – and they can then expect either to be fired to forced into retirement. In Figure 4 we let wages be rising linearly in age so that w = 1+0.002(age – 20). Note that age=20 gives a wage of 1 and an age of 65 gives a wage of 1.09. This implies that a 65 year old worker is 9% more costly for a given level of productivity – her/his unit labour costs are 9% higher – than a worker of age 20.

shifting of the firing thresholds. A comparisons of the left-hand side of Figure 1 with Figure 4 show this to be the case.

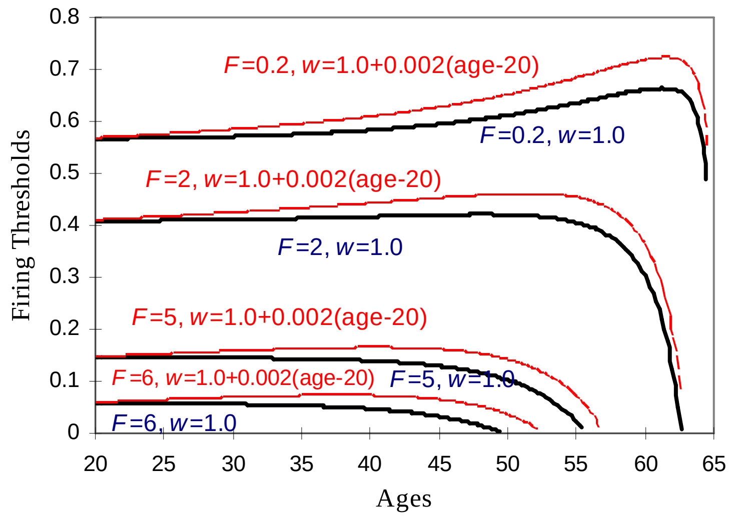

Figure 5.The effect of age on the firing thresholds with different firing costs and linear wage function (firing-only scenario). Ages are equal to (65–*T*). Other parameters: σ=0.20, ρ=0.10, θ=0.7, η=0.02, *w*=1, *H* = 0.083, and *t*=0. The bolder lines show the case of constant wages.

The figure shows that when wages rise with age, the previous results hold but it takes a higher level of firing costs to make the firm choose to fire the younger workers since the older workers are now more costly to employ. Numerical simulations show that with F=5 or more, the firm will fire the younger workers first, although the differences are small among young workers.

In the second figure we let firing costs *F* be rising in workers' age using a logistic function.[12] A logistic function is used:

$$F = 0.2 + \frac{1.3}{1 + \exp\left(-0.5 \cdot \left(age - 30\right)\right)}. \tag{20}$$

According to the function, *F* = 0.2 when the worker is young and *F* =0 .2+1.3 = 1.5 when he is old, the central/reflective point being around age=30 years, and the transitional speed from *F*=0.2 to *F*=1.5 is controlled by the parameter "0.5", the bigger the transitional speed parameter, the faster it is transiting from 0.2 to 1.5. The firing cost is barely changed after age=38 for employees as shown in the figure below.

[12] Appendix E has figures when firing costs rise linearly in workers' age.

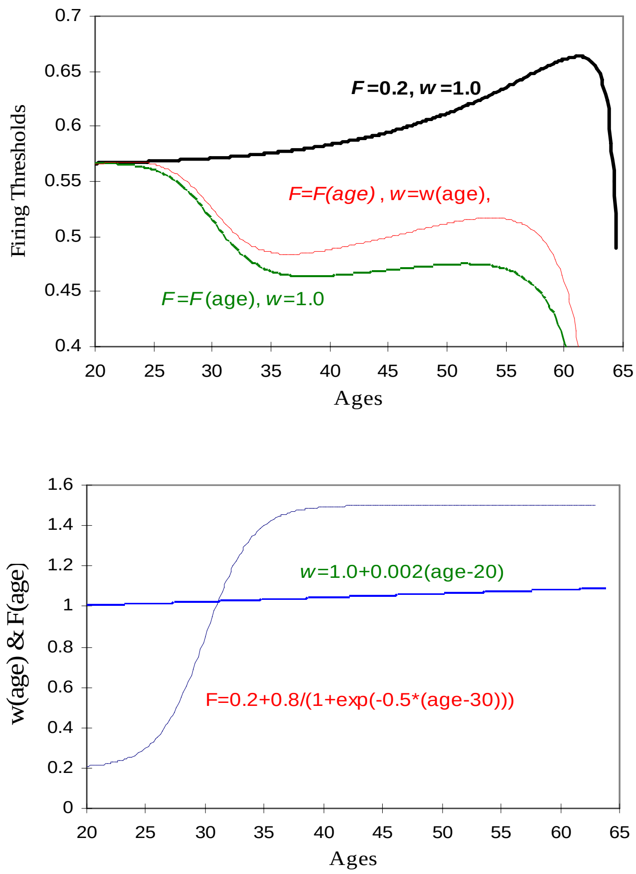

Figure 6.The effect of age on the firing thresholds with $F = 0.2 + 1.3 \big/ \left(1 + e^{-0.5(\text{age}-30)}\right)$ and $w = 1 + 0.002(\text{age} - 20)$ (firing-only scenario). Ages are equal to (65–*T*). Other parameters: σ=0.20, ρ=0.10, θ=0.7, η=0.02, *w*=1, *H* = 0.083, and *t*=0. The bolder lines show the case of constant firing costs and wages.

The results show that as the firing costs rise with age and wages remaining constant, the firm tends to fire the young first as the young are least protected by the firing costs in economic downturns. This reinforces our previous results.[13] Moreover, if both wages and firing costs are rising in age, the firm would fire the young and/or the old first, depending on the actual functional forms of *w*(age) and *F*(age) and keep the incumbent middle-aged employees. The old are costly to keep (higher wages) and the young are less protected by the firing costs than the old.

[13] Note that as the *F*(age) function rarely changes after age 38 and the effect of the firing options hence dominate when the firing costs are still relatively small – albeit at a slower increasing rate for the firing

7. Conclusions

We have provided an explanation for the observed positive relationship between youth unemployment and the cost of firing workers. When the cost of firing workers is high, firms only fire when the present discounted value of future losses at the firing margin is high, in which case they gain little by postponing the firing decision in the hope that productivity will recover. The young workers are then the first to go. In contrast, when the cost of firing workers is low, the present discounted value of future losses is small at the firing margin and firms can choose to wait in the hope of a recovery. In this case they may choose to fire the older workers first since the younger ones have longer expected tenure and are more likely to be around when productivity recovers. In contrast, for all plausible parameter values, firms will choose to hire and train the young workers first since they have longer expected tenure and a larger firing option.

It follows, that the effects of employment-protection legislation are likely to depend on the age structure of the population. Such legislation is most effective in deterring the dismissal of mature workers and, as a result, is more likely to lead firms to dismiss the younger ones. The effect arises for the sole reason that the value of the firing option is decreasing in both the level of firing costs and in the age of the worker. Furthermore, this effect is reinforced when older workers are more likely to quit due to higher expected benefits in a defined-benefit pension schemes or the accumulation of savings in a defined-contribution scheme. Intuitively, firing the young increases the value of the firm by more than firing the old – it is a better investment – because it is less likely that they would have left the firm on their own accord. Rising age-firing

thresholds when compared with the one in the case of a constant F=0.2 and wage =1. Thus, the firm costs profiles also make firms fire the younger workers first, while rising age-wage profiles have the opposite effect of making firms fire the older workers first since they are more costly.

tends to fire the old before the middle-age workers when F depends on age and w is constant.

Appendix A:

As workers live forever (T=∞), equation (8) in the text is reduced to

$$(\rho + \lambda)v = \eta_Y Y v_Y + \frac{1}{2}\sigma^2 Y^2 v_{YY} . \tag{A1}$$

(A1) is a homogenous equidimensional linear differential equation and is easily solvable. The solutions to (A1) are:

$$v_0 = A_1 Y^{\beta_1} + A_2 Y^{\beta_2} , \tag{A2}$$

where A_1 and A_2 are coefficients and β_1 and β_2 are the roots of the following characteristic equation,

$$\frac{1}{2}\sigma^2 \beta(\beta - 1) + \eta_Y \beta - (\rho + \lambda) = 0 , \tag{A3}$$

and β_1 is positive and β_2 is negative,

$$\beta_1 = \frac{1}{2} - \frac{\eta_Y}{\sigma^2} + \sqrt{\left(\frac{\eta_Y}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\rho + \lambda)}{\sigma^2}} > 0 , \tag{A4}$$

$$\beta_2 = \frac{1}{2} - \frac{\eta_Y}{\sigma^2} - \sqrt{\left(\frac{\eta_Y}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\rho + \lambda)}{\sigma^2}} < 0 . \tag{A5}$$

The hiring and firing solutions for v_0 are

$$v_H = A_1 Y^{\beta_1} \text{ for hiring option,} \tag{A6}$$

$$v_F = A_2 Y^{\beta_2} \text{ for firing option.} \tag{A7}$$

These are equations (10.1) and (10.2) in the text respectively.

Appendix B:
Derivation of Equations (12.1) and (12.2)

We know that if workers are expected to have infinite lives, the hiring and firing options approach $A_1 Y^{\beta_1}$ and $A_2 Y^{\beta_2}$ respectively. Thus, the first guess for the solutions to equation (8) in the text would be

$$v^G(Y,t) = Y^\beta z(Y,t) . \tag{B1}$$

Differentiating (B1) gives

$$v_Y^G = \beta Y^{\beta-1} z + Y^\beta z_Y ,$$

$$v_{YY}^G = \beta(\beta - 1) Y^{\beta-2} z + 2\beta Y^{\beta-1} z_Y + Y^\beta z_{YY} ,$$

$$v_t^G = Y^\beta z_t .$$

Substituting into equation (8) in the text gives

$$\left[\frac{1}{2}\sigma^2\left[\beta(\beta - 1)z + 2\beta Y z_Y + Y^2 z_{YY}\right] + \eta_Y(\beta z + Y z_Y) + z_t - (\rho + \lambda)z\right] Y^\beta = 0$$

or

$$\frac{1}{2}\sigma^2\left[\beta(\beta - 1)z + 2\beta Y z_Y + Y^2 z_{YY}\right] + \eta_Y(\beta z + Y z_Y) + z_t - (\rho + \lambda)z = 0 .$$

Rearranging gives

$$\left[\frac{1}{2}\sigma^2\beta(\beta-1)+\eta_Y\beta-(\rho+\lambda)\right]z+\frac{1}{2}\sigma^2\left[Y^2z_{YY}+2\left(\beta+\frac{\eta_Y}{\sigma^2}\right)Yz_Y+\frac{2}{\sigma^2}z_t\right]=0. \quad \text{(B2)}$$

The first terms in the first bracket are equal to zero automatically due to the characteristic equation of equation (A3) in Appendix A. With the assumption that the solutions of options have the same components as the ones with infinite maturity, Y^β, the functions, $z(Y,t)$, then follow a Convection-Diffusion type partial differential equation:

$$Y^2z_{YY}+2\left(\beta+\frac{\eta_Y}{\sigma^2}\right)Yz_Y+\frac{2}{\sigma^2}z_t=0. \quad \text{(B3)}$$

It is time to get rid of the Y and Y^2 terms. Let

$$Y=e^y, \qquad -\infty<y<\infty, \ \frac{1}{2}\sigma^2t=\frac{1}{2}\sigma^2T-\tau,$$

where T is a constant. Then we have

$$z_y=Yz_Y, \ z_{yy}=Y^2z_{YY}+Yz_Y, \text{ and } \frac{1}{2}\sigma^2z_\tau=-z_t.$$

Substituting into (B3) gives

$$z_{yy}+2\left(\beta+\frac{\eta_Y}{\sigma^2}-\frac{1}{2}\right)z_y-z_\tau=0. \quad \text{(B4)}$$

The boundary and conditions for options, equations (9.1) and (9.2) in the text become

$$z(-\infty,\tau)=0, \text{ for hiring options,} \quad \text{(B5.1)}$$

$$z(\infty,\tau)=0, \text{ for firing options.} \quad \text{(B5.2)}$$

Substituting the values of betas, β_1 and β_2, of equations (A4) and (A5) in Appendix A into (B4) gives

$$z_{yy}+2\sqrt{\alpha}z_y-z_\tau=0, \text{ for hiring options,} \quad \text{(B6)}$$

$$z_{yy}-2\sqrt{\alpha}z_y-z_\tau=0, \text{ for firing options,} \quad \text{(B7)}$$

where $\alpha=\left(\frac{\eta_Y}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2(\rho+\lambda)}{\sigma^2}$.

Hiring options

We can simplify (B6) by setting

$$x=y+2\sqrt{\alpha}\tau, \ \bar{\tau}=\tau.$$

Note that $\bar{\tau}$ is the same as τ. To rewrite (B6) in terms of $(x,\bar{\tau})$ we use the chain rule

$$z_\tau=z_xx_\tau+z_{\bar{\tau}}\bar{\tau}_\tau=2\sqrt{\alpha}z_x+z_{\bar{\tau}},$$

$$z_y=z_xx_y=z_x, \text{ and } z_{yy}=z_{xx}.$$

Substituting into (B6) gives

$$z_{xx}=z_{\bar{\tau}}. \quad \text{(B8)}$$

A new variable that depends only on x and $\bar{\tau}$ is often used to solve the above partial differential equation:

$$\xi=\frac{x}{\sqrt{\bar{\tau}}}, \quad \text{(B9)}$$

so that $z(x,\bar{\tau})=u(\xi)$. Differentiating shows that

$$z_{\bar{\tau}} = -\frac{1}{2\bar{\tau}}\xi u'(\xi),\ z_{xx} = \frac{1}{\bar{\tau}}u''(\xi).$$

Substituting into equation (B8) gives the following second-order ordinary differential equation:

$$u''(\xi) + \frac{1}{2}\xi u' = 0,\quad -\infty < \xi < \infty, \tag{B10}$$

The boundary condition of (B5.1) becomes the following equation:

$$u(-\infty) = 0, \text{ for hiring options,} \tag{B11}$$

Separating the variables, (B10) becomes

$$u'(\xi) = B_1 e^{-\xi^2/4},$$

where B_1 is unknown constant. Integrating gives

$$u(\xi) = B_1 \int_{-\infty}^{\xi} e^{-s^2/4} ds + C_1, \tag{B12}$$

where C_1 is an unknown constant. Applying the boundary condition for hiring options (B11) gives

$$\lim_{\xi \to -\infty} u(\xi) = C_1 = 0.$$

Substituting into (B12) gives

$$u(\xi) = B_1 \int_{-\infty}^{\xi} e^{-s^2/4} ds.$$

It is convenient to make the change of variable $s = \sqrt{2}\varpi$, so that

$$u(\xi) = B_1\sqrt{2}\int_{-\infty}^{\xi/\sqrt{2}} e^{-\varpi^2/2} d\varpi = A_1 \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\xi/\sqrt{2}} e^{-\varpi^2/2} d\varpi. \tag{B13}$$

where $A_1 = B_1 2\sqrt{\pi}$. Substituting (B13) into (B1) and using the facts of $Y = e^y$, $z(x,\bar{\tau}) = u(\xi)$, $\xi = \frac{x}{\sqrt{\bar{\tau}}}$, $\frac{1}{2}\sigma^2 t = \frac{1}{2}\sigma^2 T - \tau$, $x = y + 2\sqrt{\alpha}\tau$, and $\bar{\tau} = \tau$ gives the hiring options v_H^G,

$$v_H^G(Y,t) = A_1 Y^{\beta_1} N(d_1), \tag{B14}$$

where $d_1 = \dfrac{\ln Y + \sigma^2 (T-t)\sqrt{\left(\frac{\eta_Y}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\rho+\lambda)}{\sigma^2}}}{\sigma\sqrt{T-t}}$ and $N(d) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d} e^{-\varpi^2/2} d\varpi,.$

Firing options

In a similar way, we can obtain the firing options. We can simplify (B7) by setting $x = y - 2\sqrt{\alpha}\tau$ and $\bar{\tau} = \tau$.

$$z_{xx} = z_{\bar{\tau}}. \tag{B15}$$

A new variable $\xi = \frac{x}{\sqrt{\bar{\tau}}}$ is used to solve the above partial differential equation so that $z(x,\bar{\tau}) = u(\xi)$. Differentiating and substituting into (B15) gives the following simple second order ordinary differential equation:

$$u''(\xi) + \frac{1}{2}\xi u' = 0,\quad -\infty < \xi < \infty.$$

Separating the variables, the above equation becomes

$$u'(\xi) = B_2 e^{-\xi^2/4},$$

where B_2 is unknown constant. Integrating gives

$$u(\xi) = B_2 \int_{-\infty}^{\xi} e^{-s^2/4} ds + A_2, \tag{B16}$$

where A_2 is an unknown constant. The boundary condition of (B5.2) becomes the following equation:

$$u(\infty) = 0, \quad \text{for firing options,} \tag{B17}$$

Applying the boundary condition for hiring options (B16) gives

$$\lim_{\xi \to \infty} u(\xi) = 2\sqrt{\pi} B_2 + A_2 = 0 \Rightarrow B_2 = -\frac{A_2}{2\sqrt{\pi}}.$$

Substituting the above relationship back into (C16) gives

$$u(\xi) = A_2 \left(1 - \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{\xi} e^{-s^2/4} ds \right).$$

It is convenient to make the change of variable $s = \sqrt{2}\varpi$, so that

$$u(\xi) = A_2 \left(1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\xi/\sqrt{2}} e^{-\varpi^2/4} d\varpi \right) = A_2 \frac{1}{\sqrt{2\pi}} \int_{\xi/\sqrt{2}}^{\infty} e^{-\varpi^2/2} d\varpi = A_2 \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\xi/\sqrt{2}} e^{-\varpi^2/2} d\varpi.$$

Thus, the firing options v_H^G becomes

$$v_F^G(Y,t) = A_2 Y^{\beta_2} N(-d_2), \tag{B18}$$

where $d_2 = \dfrac{\ln Y - \sigma^2 (T-t)\sqrt{\left(\dfrac{\eta_Y}{\sigma^2} - \dfrac{1}{2}\right)^2 + \dfrac{2(\rho+\lambda)}{\sigma^2}}}{\sigma\sqrt{T-t}}$ and $N(d) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^{d} e^{-\varpi^2/2} d\varpi$..

Appendix C:
Derivation of Equations for $\partial v^G / \partial Y$

Differentiating hiring and firing options – defined by (12.1) and (12.2) – with respect to *Y* gives

$$\frac{\partial v_H^G}{\partial Y} = A_1 \beta_1 Y^{\beta_1 - 1} N(d_1) + A_1 Y^{\beta_1} N_Y(d_1), \tag{C1}$$

$$\frac{\partial v_F^G}{\partial Y} = A_2 \beta_2 Y^{\beta_2 - 1} N(-d_2) + A_2 Y^{\beta_2} N_Y(-d_2). \tag{C2}$$

Differentiation of the integral, $N(d)$, involves a parameter. Such differentiation can be obtained by using *Leibnitz's rule*. Suppose a function

$$\varphi(x) = \int_{a(x)}^{b(x)} f(x,s) ds, \tag{C3}$$

where *f* is such that the integration cannot be effected analytically. Using calculus gives

$$\varphi_x(x) = \int_{a(x)}^{b(x)} \frac{\partial f(x,s)}{\partial x} ds + f(x, b(x)) b_x(x) - f(x, a(x)) a_x(x). \tag{C4}$$

Applying (C4) to the differentiation of $N(d_1)$ and $N(-d_2)$ gives

$$N_Y(d_1) = \frac{e^{\frac{-\left[\ln Y + \sigma^2 (T-t)\sqrt{\alpha}\right]^2}{2\sigma^2 (T-t)}}}{\sigma Y \sqrt{2\pi(T-t)}}, \tag{C5}$$

$$N_Y(-d_2) = -\frac{e^{\frac{-\left[-\ln Y + \sigma^2 (T-t)\sqrt{\alpha}\right]^2}{2\sigma^2 (T-t)}}}{\sigma Y \sqrt{2\pi(T-t)}}. \tag{C6}$$

where $\alpha = \left(\frac{\eta_Y}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\rho+\lambda)}{\sigma^2}$.

Appendix D:

Equation (8) in the text can be solved numerically by finite different method. To compare the numerical results of finite different method with the analytical solutions in the Appendix B, we use the simple and robust explicit finite difference method, which is widely used in the pricing of derivatives.

For firing options with maturity *T*, the boundary condition is $v^G(\infty, t) = 0$ and $v^G(0,t) = \max[-(aY - bw) - F, 0]$, where $a = \left(1 - e^{-(\rho+\lambda-\eta_Y)(T-t)}\right)/(\rho + \lambda - \eta_Y)$, $b = \left(1 - e^{-(\rho+\lambda)(T-t)}\right)/(\rho+\lambda)$, and F the firing costs. The terminal condition is $f(Y,T) = 0$. The condition of $v^G(Y,t) = \max\left[-(aY - bw) - F, v^G\right]$ is checked for every t since it is a free-boundary condition in a sense that the firing option can be exercised at any time.

Equation (8) in the text,

$$(\rho + \lambda) v^G = \eta_Y Y v_Y^G + \frac{1}{2}\sigma^2 Y^2 v_{YY}^G + v_t^G, \tag{D1}$$

can be approximated by the following grids.[14] Let $v^G(t,Y) \equiv v_{i,j}$,

$$\frac{\partial v^G}{\partial Y} = \frac{v_{i+1,j+1} - v_{i+1,j-1}}{2\Delta Y}$$

$$\frac{\partial^2 v^G}{\partial Y^2} = \frac{v_{i+1,j+1} + v_{i+1,j-1} - 2v_{i+1,j}}{\Delta Y^2}$$

$$\frac{\partial v^G}{\partial t} = \frac{v_{i+1,j} - v_{i,j}}{\Delta t}$$

Substituting into (D1) gives

[14] For a similar algorithm in derivative pricing, see Brennan and Schwartz (1978).

$$\frac{v_{i+1,j}-v_{i,j}}{\Delta t}+\eta_Y j\Delta Y\left(\frac{v_{i+1,j+1}-v_{i+1,j-1}}{2\Delta Y}\right)$$
$$+\frac{1}{2}\sigma^2 j^2\Delta S^2\left(\frac{v_{i+1,j+1}+v_{i+1,j-1}-2v_{i+1,j}}{\Delta Y^2}\right)=(\rho+\lambda)v_{i,j} \tag{D2}$$

Rearranging gives

$$v_{i,j}=a_j^* v_{i+1,j-1}+b_j^* v_{i+1,j}+c_j^* v_{i+1,j+1} \tag{D3}$$

where

$$a_j^*=\frac{1}{1+(\rho+\lambda)\Delta t}\left(-\frac{1}{2}\eta_Y j\Delta t+\frac{1}{2}\sigma^2 j^2\Delta t\right)$$

$$b_j^*=\frac{1}{1+(\rho+\lambda)\Delta t}\left(1-\sigma^2 j^2\Delta t\right)$$

$$c_j^*=\frac{1}{1+(\rho+\lambda)\Delta t}\left(\frac{1}{2}\eta_Y j\Delta t+\frac{1}{2}\sigma^2 j^2\Delta t\right)$$

The firing thresholds calculated from above algorithm are shown in figure D1, together with the ones in figure 1. The results show that the analytical solutions are good approximations to the real thresholds.

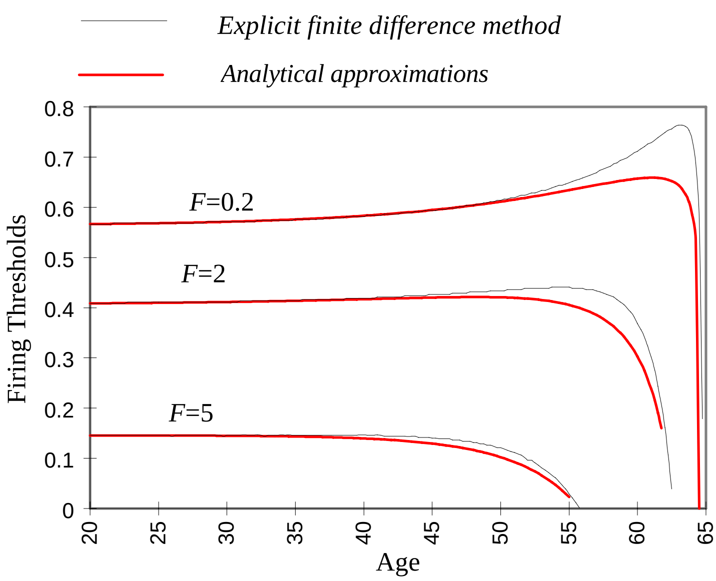

Figure D1. The comparison of explicit finite difference method and numerical approximations. All parameters are the same as in figure 1 in the text.

Appendix E:

We consider the alternative to having firing costs rise less steeply as the worker sages by redrawing Figure 6 redrawn for the case of firing costs rising linearly in workers' age: $F = 0.2 + 0.025(age - 20)$.

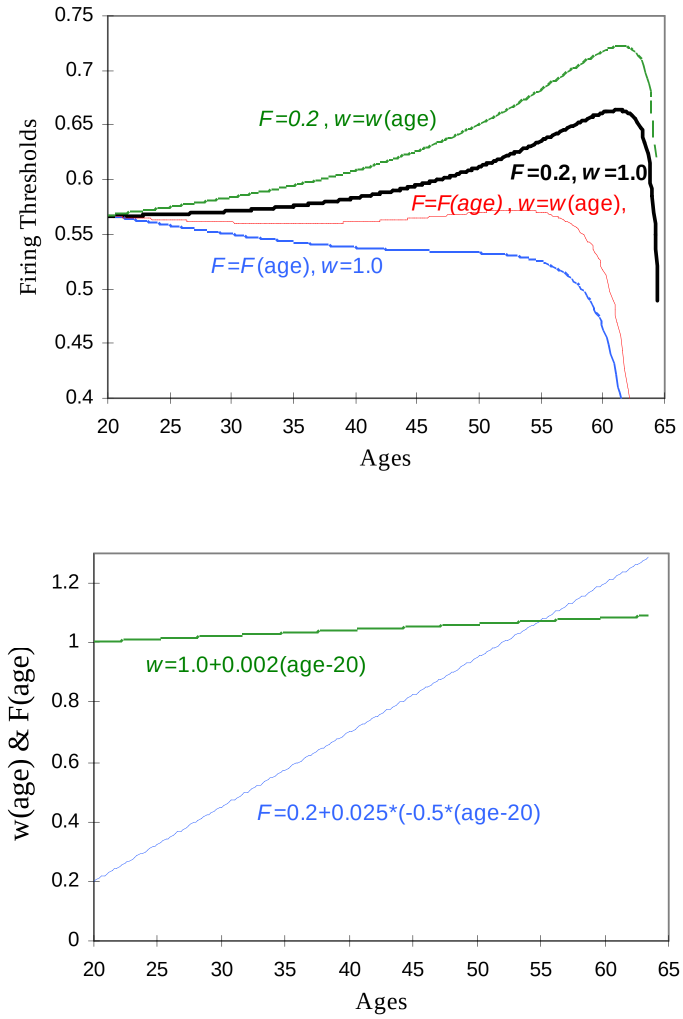

Figure 6.The effect of age on the firing thresholds with $F = 0.2 + 0.8/\left(1 + e^{-0.5(\text{age}-30)}\right)$ and $w = 1 + 0.002(\text{age} - 20)$ (firing-only scenario). Ages are equal to (65–T). Other parameters: σ=0.20, ρ=0.10, θ=0.7, η=0.02, w=1, H = 0.083, and t=0. The bolder lines show the case of constant firing costs and wages.

The results are qualitatively unchanged so that the firm will fire the younger workers first when firing costs are rising in age. Moreover, if both wages and firing costs are rising in age, the firm would fire the young and/or the old first, depending on the steepness of w(age) and F(age) and keep the incumbent middle-aged employees. The old are costly to keep (higher wages) and the young are not protected by the firing costs as the old.

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