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Unifying prospective and retrospective interval-time estimation: A fading-Gaussian activation-based model of interval-timing

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Abstract

Hass and Hermann (2012) have shown that only variance-based processes will lead to the scalar growth of error that is characteristic of human time judgments. Secondly, a major meta-review of over one hundred studies (Block et al., 2010) reveals a striking interaction between the way in which temporal judgments are queried and cognitive load on participants’ judgments of interval duration. For retrospective time judgments, estimates under high cognitive load are longer than under low cognitive load. For prospective judgments, the reverse pattern holds, with increased cognitive load leading to shorter estimates. We describe GAMIT, a Gaussian spreading-activation model, in which the sampling rate of an activation trace is differentially affected by cognitive load. The model unifies prospective and retrospective time estimation, normally considered separately, by relating them to the same underlying process. The scalar property of time estimation arises naturally from the model dynamics and the model shows the appropriate interaction between mode of query and cognitive load.

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1. Introduction

Extensive empirical evidence (Gibbon, 1977; Gibbon & Allan, 1984; Matell & Meck, 2000; Meck, 2005) suggests that time-estimation errors in interval times grow approximately linearly with the size of the estimate. Known as the scalar property of time estimation, this sets a hard constraint on the nature of the underlying processes involved in time estimation (Hass & Herrmann, 2012) and remains the sine qua non of time-estimation models.

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Secondly, in a careful meta-analysis of well over one hundred studies, Block et al. (2010) established that human adults’ perception of the passage of time differs according to whether they are forewarned that they will need to make a timing judgment, and are therefore actively attending to its passage (prospective time estimation), or whether they are required to make an unexpected, after-the-fact judgment of the passage of time (retrospective time estimation). And finally, this difference is heavily modulated by cognitive load, showing a classic cross-over interaction in which either prospective or retrospective judgments are longer depending on whether the participant experiences high or low cognitive load (Figure 1).

We will show that a model of time perception based on the idea of sampling a fading-Gaussian activation trace, GAMIT, naturally captures all three of these critical properties of interval time estimations by considering not only the amount of activation decay of the Gaussian, but also the rate at which decay is occurring.

1.1 An overview of existing models of interval timing

There are three major paradigms for interval-time judgments: (1) pacemaker-accumulator models, (2) multiple oscillator-coincidence detector models (also sometimes called timestamp models), and (3) memory or neural process models. The first class of models relies on an internal pacemaker that emits regular, short pulses that are counted by an accumulator. The number of pulses stored in the accumulator gives the measure of the time that has passed (Church, 1984; Gibbon et al., 1984; Wearden, 1991, 2001; Taatgen et al., 2007). A second class of models relies on multiple neuronal oscillators with coincidence detectors associating particular patterns of firing with given time intervals, effectively time-stamping when an event occurs (Church & Broadbent, 1990; Matell & Meck, 2000; Miall, 1989). An alternative type of oscillator-based timing model (e.g., Brown et al., 2000) assumes that some representation of the state of an already-running set of oscillators (started, say, at the birth of the individual), is associated with each event in memory, in essence, as one of the features of the event. The third class of models involves recovering the passage of time from a neural process that is decaying (Lewis & Miall, 2006; Staddon & Higa, 1999) or increasing (Reutimann et al., 2004). Here, the current state or change in state of the activation trace allows the system to recover the passage of time.

1.2 Interval timing and the scalar property

Interval timing operates in the range from half a second to several minutes. Here humans and other animals show very similar abilities. The scalar property or time-scale invariance (Gibbon, 1977) states that the width of this distribution is directly proportional to the length of the interval. So, for example, the standard deviation for a distribution of estimates of an interval of 2X seconds will be (approximately) twice that for an interval of X seconds. This effect is very widely replicated with humans, rats and pigeons (see Gibbon & Allan, 1984; Gibbon et al., 1997; Matell & Meck, 2000; Meck, 2005). Although some studies report a greater than linear increase of the timing errors (reviewed in Hass et al., 2008; Gibbon et al., 1997; Grondin, 2001).

No model that we are aware of accounts for the scalar property as an unavoidable consequence of the way the timing mechanism works (Hass & Hermann, 2012; Hass et al., 2008). For example, models based on repetitive clock-like processes have less intrinsic variability than predicted by the scalar property and must introduce assumptions as to why the cognitive system cannot use these more precise quantities. Hass and Hermann use information theoretic arguments to show how the scalar property places several important restrictions on the nature of any interval timing mechanism. In particular, they show that, in order to display scalar error profiles, the neural process underlying time perception must be based on a measure of growing variance in the system. Power law decay functions found in memory-decay models would give rise to more than linear growth in error while the errors in accumulators and oscillators grow too slowly. Accumulator models base their estimates on mean number of accumulated ticks or oscillations. However, according to the Central Limit Theorem, such estimates have errors that grow with the square root of the total. Only with logarithmic decay does a constant error around activation values convert to a scalar error in magnitude.

Accumulator models cannot account for the scalar property of time without positing a secondary process that modifies the shape of the error distribution (Hass & Hermann, 2012). Gibbon (1977) acknowledges this problem for the original Scalar Expectancy Theory (SET) pacemaker-accumulator model. In SET, the pacemaker is a Poisson process and variance in a cumulative Poisson process grows according to the square root. Gibbon et al. (1997) get around this by attributing the error primarily to a multiplicative factor associated with the comparison of accumulated estimates and their counterparts in memory, relying on a mathematical argument by Gibbon (1992). Decisions as to whether the clock has reached a given value are performed by seeing if the ratio of the accumulated value and the valued stored in memory is within a certain threshold. This ratio induces the scalar property and is
central to permitting SET to fit the empirical data. However, no justification is given for why this calculation has to be done using a ratio when comparisons of the absolute accumulator magnitudes are clearly possible and would permit the cognitive system to make temporal judgments of greater accuracy. As Staddon and Higa (1999) observe, the assumptions behind SET are far from parsimonious and the neural mechanisms that could support it are unclear.

In multiple oscillator models (Church & Broadbent, 1990; Miall, 1989) timing is measured by a large array of neuronal oscillators of different frequencies. An event starts all the oscillators simultaneously and at the end of the interval a coincidence detection mechanism learns which oscillators are in phase with each other. On future trials this same subset of the oscillators will also be in phase after the same amount of time has passed, allowing this signal to be used as timing mechanism. However, in general, this signal does not show the necessary scalar properties. In Miall’s Beat Frequency model, the distribution of firing was not normally distributed, having a sharp peak at the target time and smaller peaks at the major harmonics of the fundamental interval. In addition, the width of the peak was not proportional to the length of the interval. Matell and Meck’s (2000) Striatal Beat Frequency model tried to address these problems. They made a sequence of modifications to Miall’s model that induced the scalar property. This required globally varying oscillators to remain perfectly correlated with each other.

A third class of model is based on memory decay and neural activation. Activation decay and growth processes are ubiquitous and well understood and can account for evidence that timing and memory use the same cognitive resources (Fortin, 1999; Fortin & Rousseau, 1997) and both recruit the dorso-lateral prefrontal cortex (Genovesio et al., 2006; Wager & Smith, 2003). However, derivation of the scalar property is not always straightforward in these models. In the Multiple Time Scales model (MTS; Staddon & Higa, 1999), a series of leaky integrators with power law decay must be carefully chained together to approximate the required logarithmic function. The Temporal Context Model (TCM; Shankar & Howard, 2010) is built from many leaky integrators using complex dynamics.

By contrast, Reutimann et al. (2004) use a single climbing neuronal trace that reaches a threshold at the expected end of an interval. Single cell recordings in the inferotemporal cortex of monkeys have found neurons with the appropriate time-dependent firing rates (Kojima & Goldman-Rakic, 1982; Komura et al., 2001; Leon & Shadlen, 2003). Learning of new intervals occurs via Hebbian learning within the adaptation process, such that neuronal firing reaches threshold at an earlier or later time. This threshold varies according to a normal distribution around a constant level. The interaction of the linearly increasing trace and the threshold gives rise to the scalar property. Advantages of this are that it is built on a single mechanism using well-understood principles of synaptic plasticity and the decision rule is built into the model itself. But the scalar property derives primarily from the gaussian nature of the threshold, which appears to be an arbitrary choice to fit the data. Recent work by Simen et al. (2011) extends this idea.

1.3 Retrospective and prospective time estimation

Time judgments can be made with or without prior notice. In retrospective time estimation an individual is asked to estimate how long ago an event occurred without prior warning that they would have to do so. By contrast, in prospective time estimation the individual knows in advance that they will be asked to estimate the time that has elapsed from a particular event. Historically, these have been studied as separate phenomena. We believe that prospective and retrospective time estimation are intimately related and should not be considered as distinct phenomena.

1.4 Cognitive load in existing models.

Our estimates of time passing can be affected by whether or not we are actively attending to the passage of time and by the amount of additional cognitive load we face. Block et al. (2010) analyzed the results from over one hundred interval-timing studies and summarized their results in the graph shown in Figure 1. They found a striking interaction between the type of time judgment requested and cognitive load. High cognitive load increases your estimates in the case of retrospective timing, whereas high cognitive load decreases your estimates in the case of prospective timing. This strong interaction is puzzling for two reasons. First, as discussed above, the mere fact that there is a difference between prospective and retrospective time is a challenge to clock and timestamp models. There is no a priori reason to expect a difference between these two conditions. Secondly, the interaction with cognitive load suggests that cognitive load is not just an additive factor (e.g., damping responses across the board). This is a challenge for all existing models of interval timing.
Figure 1. The effects of cognitive load on interval timing based on a meta-analysis of 82 prospective and 31 retrospective tasks (Block et al., 2010). Duration judgment ratio is the ratio between subjective estimates of time and the actual objective time that has passed. Error bars show standard errors.

2. GAMIT: A fading-Gaussian activation-trace model of interval-timing

GAMIT is built on the assumption that our sense of time is learned through our experience of changes in the world around us. Small changes in the activation trace mean little time has passed; large changes mean a lot of time has passed. These changes allow us to interpret a fading-Gaussian activation trace associated with a particular event as the passage of time. In addition, we assume that the original activation trace generated by an event fades over time in a statistically predictable manner and that the rate of this decay is affected by cognitive load.

To implement the GAMIT model, we begin with a cluster of cortical columns. The activation in the central column corresponds to an event in the world that is registered in memory. Activation then spreads across the cortical columns as follows. If we designate the activation of the \( i^{th} \) column at time step \( t \) by \( A_i(t) \), its activation at time \( t+1 \) is determined by the following equation:

\[
A_i(t+1) = \alpha A_i(t) + \beta(A_{i-1}(t) + A_{i+1}(t)) + \xi
\]

where \( \alpha \) is the fraction of activation that remains in column \( i \) on each time step (i.e., \( \alpha = 1\)-leakage); \( \beta \) is the fraction of activation spread from each immediate neighbor of \( i \) on each time step; \( \xi \) is a noise parameter. The values of \( \alpha \) and \( \beta \) must be chosen so that the total activity over time of the system neither rapidly decreases to zero nor increases exponentially. Unless otherwise stated, we used values of \( \alpha = 0.7 \), \( \beta = 0.14952 \) and \( \xi = 0.000025 \). Initially, only the central column is activated and activation spreads over time as shown in Figure 2.

Figure 2. The activation of the initial Gaussian above the central column fades and spreads with time.
There is ample neurobiological evidence for this type of spreading-activation mechanism (e.g., Amari, 1980; Capaday et al., 2011; Grinvald et al., 1994; Grossberg, 1980; Herman et al., 1993; Koch & Segev, 1998). We argue that the cognitive system is sensitive to both $H(t)$, the maximum height of the fading Gaussian at time $t$ and $A(t)$, the total activation of the fading Gaussian at time $t$. The spreading-activation values on which time estimates are based $S(t)=H(t)+A(t)$, is the sum of these two values and provides a stable estimate of the long-term average of the underlying stochastic diffusion process.

3. Time estimates and the scalar property

Time in GAMIT is estimated by determining how much a stochastic activation trace has faded compared to a Reference Activation-Decay Curve built from a lifetime of experience. Central to our explanation of both retrospective and prospective time estimation is the assumption that, over time, we have learned a typical or “average” activation-decay curve for events and that this curve serves as a reference curve for time estimation (Figure 3; cf. Addyman et al., 2011). Errors associated with the decay of individual traces and introduced during the comparison process give rise to scalar property.

To test the scalar property, we assume that the current activation curve is decaying as shown by the red curve in the Figure 4. This curve differs slightly from the (white) Reference Activation-Decay curve. At $t = 800$, the “actual elapsed time,” a time judgment is requested. This time corresponds to an activation level of the current activation curve of $S = 0.652$. We sample from the Gaussian error distribution around $S$ and get a value of $S = 0.656$, for which
the corresponding time value on the Reference Activation-Decay Curve is \( t = 1075 \) (“perceived elapsed time”). The time-estimation error is the difference between the perceived and actual elapsed time, i.e., 275 time units.

We consider all time values, \( t \), between \( t = 1 \) and \( t = 750 \) and calculate the error, \( E \), for each of these time values as described above. Averaged over 250 runs of the program, we obtain a linear fit \( E = 0.23t \) to the data with an \( r \) of 0.99 (Figure 5). In other words, the spreading-activation decay mechanism in GAMIT naturally satisfies Hass and Herrmann’s (2012) variance requirements for scalar growth in time-estimation error and does not require the positing of any further secondary mechanisms.

**4. Modeling retrospective and prospective time judgments under cognitive load**

A key feature of our model is that the activation and sampling profiles are differentially affected by cognitive loads. To begin with, we assume that greater cognitive load causes more rapid decay of the trace activation, due primarily to global inhibition from other tasks that require encoding and storing information in memory (Figure 6). This alone allows us to explain differences in retrospective time estimation under cognitive load (Figure 7). However, in the case of prospective time estimation, when it is known ahead of time that a time judgment will be required, we further propose that the state of the spreading-activation trace will be repeatedly sampled. Sampling

Figure 6. The typical activation-decay curve, \( S(t) \), learned by experience is shown in blue. Under high cognitive load, spreading-activation falls off more quickly than under typical cognitive load. Under (very) low cognitive load (in pink) spreading-activation falls off more slowly than typical load conditions.
can be thought of in our model as “attentional saccades” to the event trace. Just as visual saccades involve a switch of visual attention, we suggest that mental saccades involve a switch of focus of attention to the trace. Just as the rate of visual saccading is interfered with by increased cognitive load (Halliday & Carpenter, 2010; Stuyven et al., 2000), we suggest that the same is true of attentional saccading. In other words, attentional resources are limited and must be distributed among the currently active tasks in working memory. Similarly, as cognitive load increases and more tasks must be processed with limited attentional resources, fewer resources (attentional saccades) are allocated to attending to the activation trace of the event whose time is to be judged (see the time-sharing hypothesis; Buhusi & Meck, 2006).

Over time the cognitive system learns a very simple association -- namely, the more the activation of a trace has changed since it was last sampled, the more time that has elapsed. In other words, small changes in activation correspond to small changes in time; large changes in activation correspond to large changes in time. This is one of the key insights to understanding the fading-Gaussian model of interval-time estimation.

4.1 Retrospective time estimation

In retrospective time judgment there is no prior announcement that a time judgment will have to be made. This means that there is no sampling of the activation trace prior to the moment when the time judgment must be made, and corresponds to what Zakay and Block (2004) refer to as “remembered duration,” since there is no on-going experience of the time interval between the moment of the stimulus event and the time when a time judgment must be made. Thus, the only cue to the amount of time that has passed is the total activation of the memory trace.

We assume that under high (low) cognitive load the spreading-activation curve falls more (less) rapidly because there is more (less) inhibition from the other concurrent tasks in working memory (Figure 6). This means that under high cognitive load there will generally be less activation in the trace than normal, which causes over-estimates of the interval length when reading from the Reference Curve (Figure 7).

Suppose that under high cognitive load, at the moment of a time judgment (e.g., t = 600), the activation value is approximately 0.66. But, in memory, there is only a stored representation of the activation curve under typical cognitive load. Based on this reference activation-decay curve (i.e., the blue curve in Figures 3, 4, and 6), an activation level of 0.66 occurs, not at t = 600, but rather, at t = 710. Thus, when asked for a (retrospective) time judgment at t = 600, we reply 710. In other words, we overestimate the amount of time that has passed under high cognitive load. This concurs with Block et al.’s (2010) finding.

Figure 7. A run of GAMIT retrospectively estimating time under high cognitive load. Under high cognitive load a retrospective interval-time judgment is judged to be longer than its actual duration.

4.2 Prospective time estimation

In prospective time estimation the participant knows ahead of time that a time judgment will be required about a particular stimulus event at some point in the future. This implies an on-going monitoring of the activation trace, a process that engenders what Zakay and Block (2004) refer to as “experiencing time”. In other words, the activation trace associated with that event will be sampled more or less frequently until the time estimation has been made. The frequency of this sampling -- what we refer to as “attentional saccading” -- depends on cognitive load. In GAMIT,
this attentional sampling is what provides information about the rate-of-change of total activation of the trace. Crucially, we assume that there is a “typical” sampling profile that defines when and how often sampling of the activation trace occurs under typical cognitive load in the context of prospective time estimation. We assume that, under high cognitive load this “attentional saccading” to the activation trace occurs less frequently because sampling requires cognitive resources and part of those resources are being diverted to additional mental activities (Figure 8).

We argue that our perception of the passage of time is intimately related to the rate of this sampling of the activation trace. An analogy is helpful here. Consider some event that unfolds over approximately 30 seconds, say, a woman walking down a street. Now, assume that we have two cameras at the scene: the first films this event at a “typical” speed of 20 frames/sec.; the second camera must film, not only this event, but simultaneously, some other event nearby. The latter camera shoots one frame of the walking woman, followed by one frame of the other event, then one frame of the woman, etc. The film of the woman walking is put together in the cutting room and, of course, contains only 10 frames/second of the woman walking. When shown both films, people will say that the flow of time is faster in 10 frames-per-second film (Eagleman, 2004). We suggest that the reason is because between each image in the 20 frames/sec case very little changes, whereas there is a much greater change between each image in the 10 frames/sec case. This observation is the key to GAMIT’s prospective time judgments under load.

4.2.1 Prospective time estimation under high cognitive load

We claim that prospective time estimations rely on, not only the amount of activation decay, but also the rate at which the activation is decaying, as measured by activation change between attentional saccades. To calculate the approximate rate of change of the decreasing activation function, a small number of recent activation changes between successive samplings of the activation curve are kept in memory. The average of these values provides an estimate of the rate at which the activation curve is falling. Over time, the cognitive system under normal cognitive load learns how much the activation trace associated with an event typically changes between attentional saccades. This value, stored in memory, we call $\Delta_{\text{Typical Load}}$. Under high cognitive load we sample the curve less often because some of the resources devoted to attention saccading are devoted to the other tasks (Figure 8). Thus, the amount of activation change between each attentional saccade, $\Delta_{\text{High Load}}$, is greater. (For low cognitive load, $\Delta_{\text{Low Load}}$ is calculated in the same way, except there is more than average sampling of the activation trace.) This is the intuition behind the definition of a time-compression (or time-dilation) factor $\Phi = \frac{\Delta_{\text{Typical Load}}}{\Delta_{\text{Current Load}}}$ for prospective time judgments. A prospective time estimate, $P$, is the retrospective time estimate, $R$, adjusted by the multiplicative time-compression/dilation factor, $\Phi$. In other words, $P = R \Phi$.

![Figure 8](image-url)  
Figure 8. The blue curve shows the evolution of activation under typical cognitive load. For prospective time estimation under typical cognitive load the frequency of sampling of this curve is shown by the blue square markers.

4.3 Simulation of Retrospective and Prospective time estimation together

In order to simulate changes in cognitive load within GAMIT, we varied the amount of activation spread to
neighboring columns, thereby causing the activation curve to fall more or less rapidly. To simulate high cognitive load conditions we decreased the value of $\beta$ to 0.14946. For lower-than-typical cognitive load, the value of $\beta$ was increased to 0.14955. The other parameters remained unchanged.

Cognitive sampling was decreased by 50% in the High Cognitive Load condition and increased by 10% in the Low Cognitive Load condition. (This asymmetry reflects the fact that cognitive load can only be decreased slightly with respect to the typical cognitive load condition, whereas it can be increased essentially without bound.) The results are shown in Figure 9. As in Block et al. (2010), an ANOVA showed that there is no significant main effect of either Cognitive Load or Retrospective-Prospective time estimation. However, again, as in Block et al., there was a highly significant interaction between the two main variables [$F(1,76) = 19.3, p < .0001, \eta^2 = 0.2$]. In short, GAMIT qualitatively reproduces the interaction between cognitive load and mean time-judgment duration ratio reported in Block et al. (2010).

![Figure 9. Performance of GAMIT on prospective/retrospective time judgments under high and low cognitive load. Results averaged over 20 runs of the program. (SEM error bars)](image)

5. General Discussion and Conclusion

We have described a "fading-Gaussian" model of interval-time estimation, GAMIT, which is based on the classic equation of spreading activation as an approximation to the underlying stochastic processes involved in the spread of information in a distributed cognitive system.

A first contribution of the model is that it takes seriously recent analyses of the scalar property of time showing that mechanisms underlying time estimation should be noisy, stochastic and based on the spread of information (Buhusi & Oprisan, 2013; Hass & Hermann, 2012).

A second contribution is that it provides a unified account of retrospective and prospective time judgments. These have traditionally been studied as separate phenomena (Block et al., 2010; Zakay & Block, 2004; but see Brown & Stubbs, 1992). Our model suggests that the same underlying mechanisms (namely, decay and sampling) operate in both contexts. What varies is the amount of sampling that occurs between the two contexts: no sampling occurs on retrospective time estimates, whereas repeated sampling occurs in the prospective time estimates. The effect of cognitive load on interval-time perception is explained by how cognitive load affects the rate of sampling of the activation trace. Thus, the GAMIT model provides a parsimonious account of interval-time judgments and places the onus on other theoretical accounts to say why two distinct timing mechanisms are necessary.

In summary, we have described a parsimonious model of interval timing that is based on ubiquitous neural and cognitive processes. It provides a unifying account of retrospective and prospective timing, captures the modulating effects of cognitive load on both prospective and retrospective timing, and in contrast to other models, intrinsically captures the scalar property of time judgments.

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