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Full Length Research Paper

Seasonality in cocoa spot and forward markets: Empirical evidence

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This paper first describes the main features of supply and demand in cocoa spot markets. A state-variable model is proposed to describe the random evolution of cocoa forward curves over time, which essentially adapts to agricultural commodities, introduced by Borovkova and Geman (2006) for energy. In contrast to most of the literature on the subject, the first state variable is not the spot price, as it combines seasonal and stochastic features and may not be observable, instead, the average value of all liquid futures contracts is a quantity devoid of seasonality and conveys a robust representation of the forward curve level. The second state variable is a quantity analogous to the stochastic convenience yield, which accounts for the random changes in the shape of the forward curve. We conduct estimation procedures for the cocoa market over the period of 1980 to 2009 and exhibit an interesting result on cocoa seasonality as well as an extension of the Samuelson effect.

Key words: Cocoa Spot markets, forward curve stochastic modeling, future curve average value.

INTRODUCTION

Cocoa originated in South America millennia ago. Archeological evidence in Costa Rica indicates that cacao, a chocolate drink named after cocoa, was drunk by Maya traders as early as 400 BC. After a Spanish expedition in 1519 brought back to Spain the recipe for cacao, cocoa was planted in Spanish territories like the Dominican Republic or Trinidad. Around 1660, France introduced cocoa to Martinique, Brazil and Grenada. England had cocoa growing in Jamaica by 1670; prior to this, the Dutch had taken over plantations in Curacao when they seized the island in 1620.

Lately, the explosion in demand brought by chocolate's affordability required more cocoa to be cultivated. Cocoa from Brazil was planted in Nigeria in 1874 and Ghana in 1879. In Cameroon, cocoa was introduced during the period 1925 to 1939. The natural habitat of cocoa trees is in the lower level of the evergreen rainforest as climatic factors, particularly temperature and rainfall, are important for optimal growth; variations in the yield of cocoa trees across years are affected more by rainfall than any other climatic factor. Today cocoa is produced in countries located in a band around the Equator, the largest producers being Cote d'Ivoire, Ghana and Indonesia as developed below.

Moreover, 95% of the world's cocoa is grown almost entirely in small farms with the whole family working together, in more than 50 countries located in Africa, Asia, the Caribbean and Latin America. In these countries, cocoa serves as the main source of cash income for millions of people whose rural livelihoods depend on it. Despite some investments made possible by the high spot prices of the marketing year 2009 to 2010, the formulation of agricultural policies in cocoa-dependent countries would benefit from a sound understanding of the price evolution in the cocoa futures market, as this one accounts for more than 70% of cocoa beans sales.

Countries recognized as cocoa producers include Ivory Coast[1] - which is the world's biggest supplier and accounts for more than 35% of global output; Ghana,

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[1]A number of Ivorian cocoa trees, planted more than 25 years ago, have already passed their peak of productivity with many farmers switching to the more lucrative rubber.

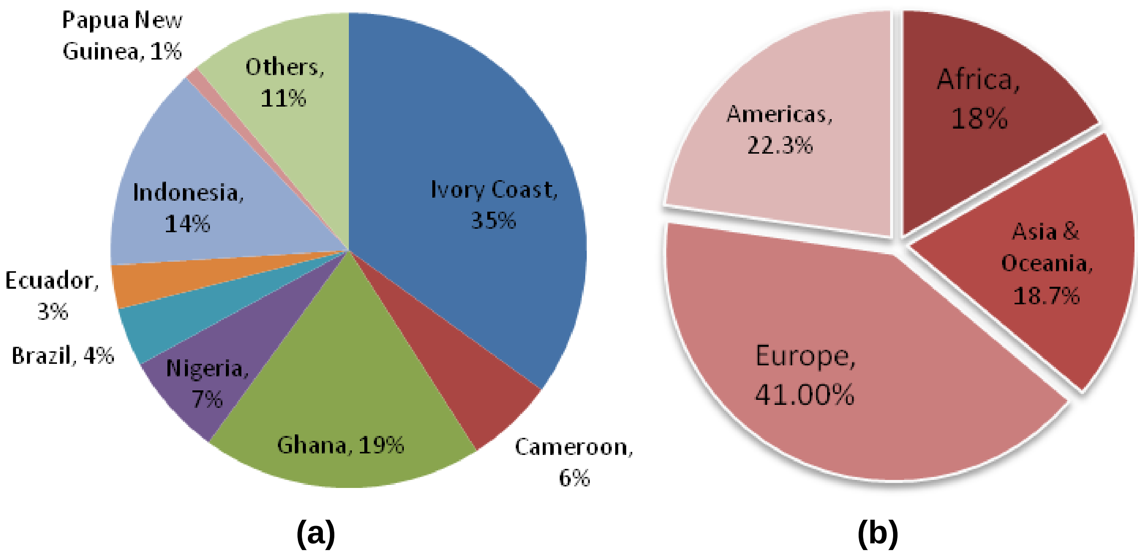


Figure 1. (a) 2008/2009 share in world production; (b) Share in world cocoa consumption (source: ICCO Annual Report, 2008/2009).

which is the second largest one; Indonesia, Cameroon, Nigeria, Brazil, Ecuador, Malaysia and others. Ivory Coast planted millions of cocoa trees during the period 1980 to 2005, thus becoming the top world producer. Using data from the International Cocoa and Coffee Association (ICCO), we display in Figure 1(a) the shares of the major cocoa producing countries. We can already observe, as it will matter for the discussion of seasonality in the next sections of the paper, that the main producers are immediately close to the Equator, in the Northern hemisphere like Ivory Coast or the Southern hemisphere like Brazil, which implies that harvests are spread over the calendar year.

In all these countries (except to a small extent for Malaysia and Indonesia), production comes from small entities with the average farm size not exceeding four hectares. Production from countries like Togo, Papua New Guinea, Dominican Republic, Peru, Mexico is individually insignificant but constitute together 10% of the world supply.

Apart from Ghana and to some extent Cameroon, where production levels have bounced back from the 1990s slowdown, the output from the other countries has been on the decline due to aging cocoa trees, lack of necessary investments in the cocoa sector and competing opportunities in the rubber and oil palm plantations (for example, in Ivory Coast). This explains why, after six years of stagnant world production during the period 2003- 2009, cocoa prices hit in July 2010 their highest level in three decades of $3500/tonne (Figure 2a).

Crinipellis perniciosa, commonly referred to as witches' broom, is a fungal cocoa disease that has for the past decade caused a decline in the production and yields of cocoa beans in Brazil. However, in Ecuador, the Latin America's second largest cocoa bean producer after Brazil, cocoa production has been on the increase due to the successful introduction of a new variety of plant resistant to the witches' broom disease. In West Africa, it is the so-called black pod which ruins the bean's quality; and a bad weather event is an intense episode of *harmattan,* the dry and dusty wind that blows from the Sahara and may jeopardize the flowering of the cocoa trees.

Production in both Indonesia and Malaysia takes place mostly on large plantations either privately or state owned. A downward trend in Malaysian cocoa production has been observed since the early 1990s, when the outbreak of disease coincided with the deterioration of the country's macro-economic conditions. In addition, farmers switched production from cocoa to more lucrative crops, such as palm trees, in response to the fall in world cocoa prices during the 1990s. The downward trend in the Malaysian cocoa sector is expected to continue due to the expansion of urban settlements. Therefore production levels observed a couple of decades ago are unlikely to be observed soon.

Government policies in Indonesia in the 1980 to 1990s have encouraged the expansion of production. Most of the increase during the last two decades was bulk cocoa coming from hybrid trees. While the expansion in Indonesia has slowed down since the late 1990s, yields in the country are still the highest among major cocoa producing countries in Asia. Like in Ghana, where producers' selling prices are linked to world market prices, cocoa farmers in Indonesia receive a high proportion of the market price and that encourages reinvestment in cocoa inputs, and in turn contributes to

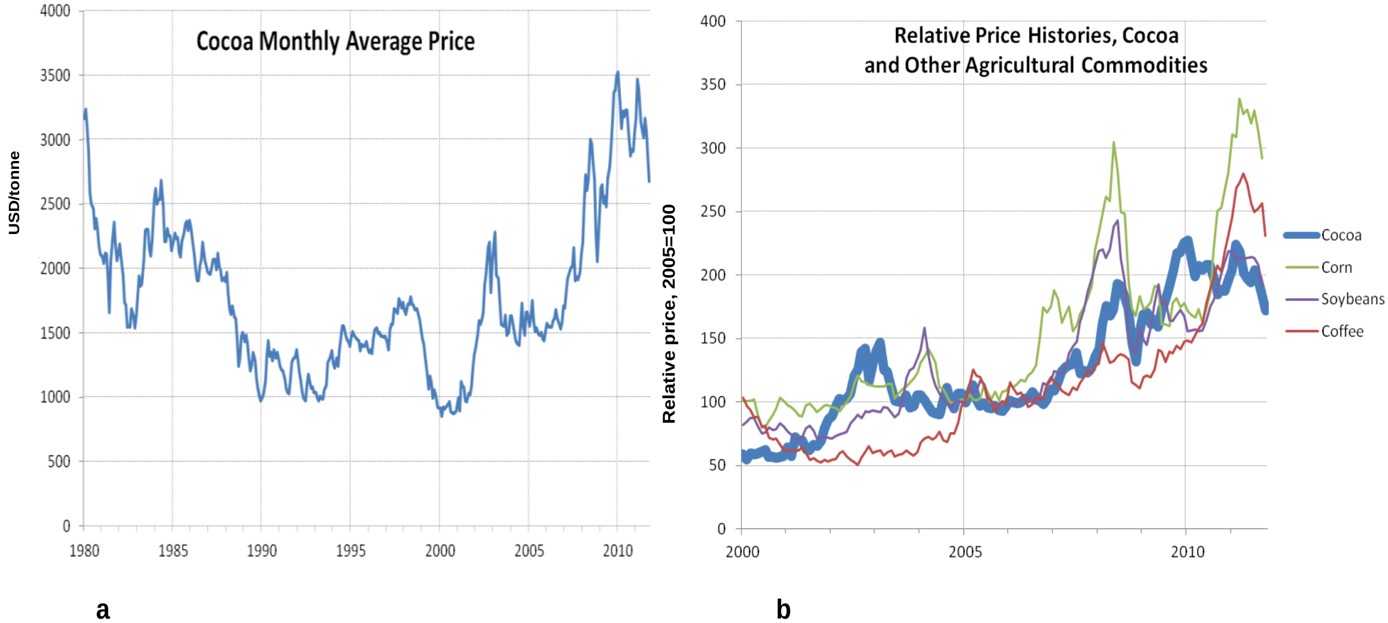


Figure 2. (a) Monthly cocoa price history, US$/tonne, 1980 to 2011(Source: World Bank); (b), Comparison of cocoa price history with other agricultural commodities, 2000 to 2011 (Source: World Bank).

improvement in yields.

There is currently an organic cocoa market but this represents a very small share of the total cocoa market. The ICCO estimates organic cocoa supplied to the world market to be less than 0.5% of total cocoa production. Production of certified organic cocoa is also estimated to be about 15,500 tonnes and sourced from countries including Madagascar, Tanzania, Uganda, Belize, Bolivia, Brazil, Costa Rica, Dominican Republic, El Salvador, Mexico, Nicaragua, Panama, Peru, Venezuela, India and Sri Lanka. Demand for organic cocoa products is growing at a very strong pace, as consumers are increasingly concerned about the safety of their food supply along with other environmental issues.

Like all agricultural commodities', cocoa prices are volatile. International prices fluctuate according to macroeconomic conditions in OECD countries (the main consumers of cocoa and cocoa products), whose annual demand has increased over the past two or three decades with an average annual growth over 3%. Cocoa is also susceptible to the vagaries of disease and weather, which can provoke wide fluctuations in world production from one year to the other. And there is a time lag between increase in demand and increase in production, as it takes at least three to five years on average for new trees to get into the production line. This time lag further contributes to the price volatility. Price volatility is magnified by trading of cocoa on futures markets where speculation is large - the volume of cocoa traded in futures contracts is ten times higher than the actual world production. The attention paid by speculators to the world's commodity markets, including agriculture, may also help to explain the co-incident spikes of many agricultural commodities including cocoa in the period 2007 to 2009 (Figure 2b) as well as the recent return of high prices.

Price volatility makes cocoa farmers vulnerable as they never know in advance what the international price will be when the harvest comes and cannot plan their production accordingly. Production prices are generally low due to lack of efficiency and fairness along the cocoa supply chain. World production of cocoa increased rapidly over the course of the 20th century, prompted by rising demand. From an annual production of less than 125,000 tonnes in the early 20th century, the annual global output has risen to 3.5 million tonnes at present, an annual average growth rate of 3.4%. Interestingly, this number is fairly similar to the growth in demand mentioned earlier, resulting in a fairly stable picture of the supply- demand balance

The remainder of the paper is as follows; A review of the literature related to agricultural commodities; description of the two state- variable model that we wish to apply to cocoa futures; the display of calibration results for the period of 1980 to 2011; conclusion.

LITERATURE REVIEW

The cocoa spot and futures markets

The International Cocoa Standards require cocoa of merchantable quality to be fermented, thoroughly dry, free from smoky beans, free from abnormal odors and any evidence of alteration. It must be reasonably free from living insects, broken beans, fragments and pieces of shell and foreign matter and reasonably uniform in size. Throughout the world, the standards against which all cocoa is measured are those of Ghana cocoa, Ghana producing the best type of cocoa. Cocoa is graded on the basis of the count of defective beans in the cut test (source: UNCTAD- information on cocoa). Nearly all cocoa sold in the spot market either from farmers' associations to merchants or from individual farmers to a chain of middle men who then sell to exporters.

Forward and futures markets

While cocoa in the spot market is immediately delivered, there is a lag between the transaction and the physical delivery in the case of a forward trade. As a brief reminder, we recall that a forward contract signed between a buyer and a seller at date 0 involves no cash flow nor any exchange of goods, but just an agreement about the 'forward price', number of dollars to be paid at a fixed maturity T by the buyer to get delivery of the commodity by the seller of the forward contract. A future contract is a similar transaction, except that now both buyer and seller interact with the exchange, whose clearing house is demanding margin deposits from both parties before any trade is placed, and *margin calls* to be paid on a daily basis by the position whose value went down from one business day to the other (namely, the buyer of the future in the case of a decline in future prices). This 'marking-to- market' of the positions through margin calls ensures the integrity of the clearing house; the existence of the exchange platform when trading futures contracts takes away the counterparty risk present in forward contracts. We can observe that, in agricultural commodity markets, not only are futures contracts quite liquid, but forward contracts as well since the old market players like Cargill or Louis Dreyfus trust each other to enter a forward transaction and gain the benefit of the secrecy of their trade that would otherwise be reflected in the change of the *open interest* (namely the number of existing contracts with a buyer and a seller at each end) posted by the Exchange on a daily basis.

Like all other futures, cocoa futures are standardized commitments in which the contract buyer agrees to take delivery from the seller a specific quantity of cocoa, namely 10 tonnes, the *contract size* for cocoa at the future price prevailing on the purchase day, for delivery date in March, May, July, September or December. These contracts allow on the one hand producers to secure ex-ante their crop revenues and on the other hand investors to get exposure to changes in cocoa prices while avoiding the hurdles of storage. There are only two places where cocoa futures contracts are traded: the London International Financial Futures Exchange (LIFFE) in the UK, and the ICE[2] futures in the United States. These organized exchanges provide the facility and trading platform that bring buyers and sellers together. The last trading day is eleven business days prior to the last business day of delivery month.

The price of the future contract at any time is the value reached by the bids of buyers and sellers and reflects the anticipation of the spot price that will prevail at the expiration of the future contract. Factors entering these bids include information about the crop outlook, whether the maturity of the contract belongs to the harvest period, as well as world cocoa inventory levels and global demand. Hence, the terminology of '*price discovery'* brought about spot prices in the future by the daily values of Futures contracts posted by the exchange

Inventory and consumption

Once harvested, fermented, dried and transported, the cocoa bean is then stored for only a very short time before it is processed in readiness for commercial consumption. The storage takes place in jute bags for brief periods; whole beans can be stored for 5 to 6 months safely. Storage extended over more than 6 months may result in loss of quality due to problems of insect infestation, mold contamination and moisture exchange between atmosphere and the beans. Other problems of prolonged storage are beans size variation. If the water content is less than 6% of the bean weight, cocoa beans become brittle, while a water content of greater than 8% may cause the risk of vapor and mold damage. Quality differences and weight, obviously, have a bearing on the trading value.

In general, the amount of world grinding of cocoa beans is used as a proxy for world cocoa consumption. Although largely produced in developing countries, cocoa consumption is concentrated in industrialized countries *(*Figure 1b*).* World demand for cocoa and cocoa products has been in a steady rise since the late 1970s *(*Figure 3a). Worldwide consumption has increased in tandem. Worldwide stocks, measured at the end of the crop year in late September, just before the major harvests of West Africa, have remained relatively steady. A useful statistic is the so-called 'stocks to grindings ratio' (called 'year-end carryover' in most agricultural commodities and constituting an important indicator for the strategies of commodity trading advisors), representing the percentage of one year's consumption remaining in stock just prior to the new harvest. This peaked in 1990 at 70% and has declined gradually to 45% since then (Figure 3b), as consumption has increased with no commensurate increase in stocks. However, 45% should still be considered ample carryover, compared with the 2011

2. The Inter Continental Exchange (ICE) was first established in Atlanta and has acquired other exchanges like the International Petroleum Exchange in London.

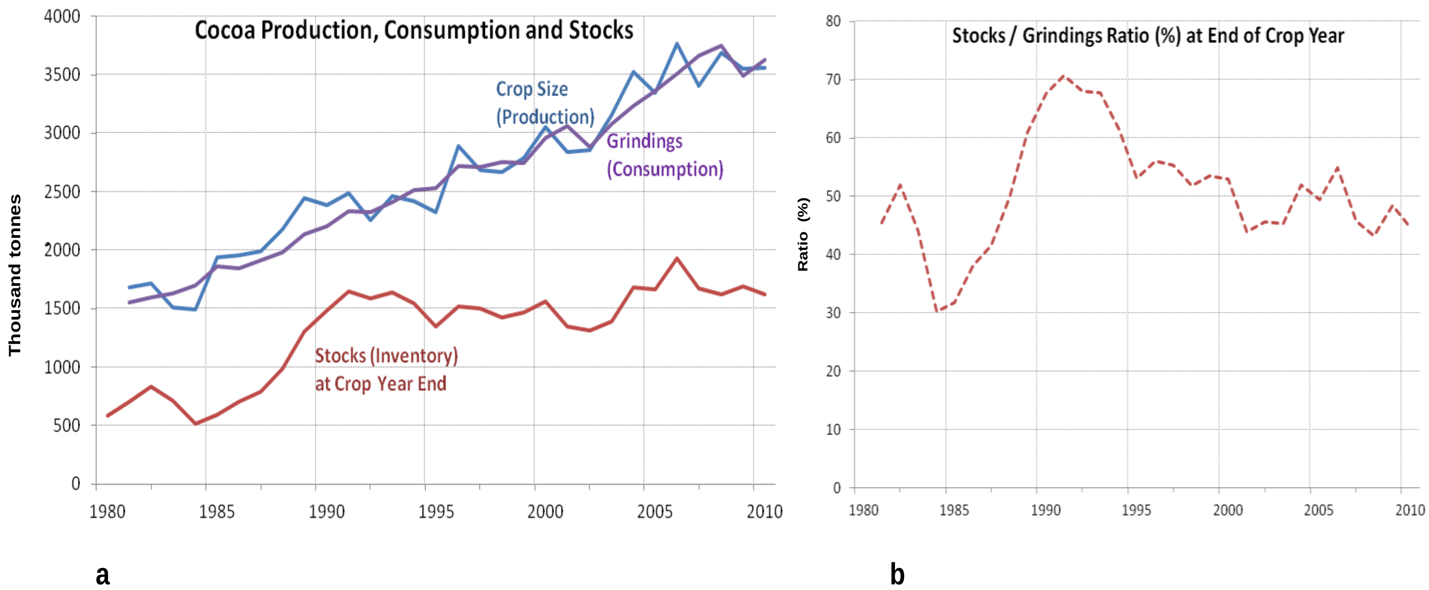


Figure 3. (a) Cocoa production, consumption ('grindings') and year-end stocks (Source: International Cocoa Organization) (b), stocks to grindings ratio, a measure of sufficient inventory.

worldwide estimates for wheat (30%), corn (16%) and rice (22%) (International Grain Council, 2011).

Besides the well- known cocoa bean, there are four other intermediate cocoa products - cocoa liquor, cocoa butter, cocoa cake and cocoa powder - and chocolate. Although the market for chocolate is the largest user of cocoa in terms of beans equivalent, intermediate products such as cocoa powder and cocoa butter are used in several areas such as the manufacturing of tobacco, soap and cosmetics.

It is worthwhile to note that the ranking of cocoa importing countries depends on the composition of the goods imported: trade is not only tracked by cocoa beans but also by semi-finished products of cocoa. The information provider global trade atlas tracks cocoa import data such as cocoa shells and cocoa paste. The Netherlands, as one of the main ports into Europe, leads in imports of beans; the United States, with significant production of cocoa complementary food products, leads in imports of powder; and France, one of the biggest chocolate markets in terms of consumption, leads in retail chocolate (Source: World Cocoa Foundation).

Prices of primary agricultural commodities used to be characterized by bursts in short- term prices and low prices on the long term. Cocoa prices, like many other commodity prices, have on the average been on the rise in the last few years as a result of factors such as excess world demand over supply following poor weather; under investment and political unrest are also persistent in some of the major producing countries.

The production of agricultural commodities continues to play a major economic role in many developing countries. Over the decades 1970 to 2000, prices of agricultural commodities on the whole have shown a pattern of long term price falls and short term price instability (International Monetary Fund, 2000). Further evidence of lack of any positive upward trend in commodity prices in the 1980s and 1990s can be found in Grilli and Yang (1988) and Cuddington (1992). Deaton (1999) shows that cocoa and coffee prices in his period of analysis were lower relative to the United States consumer price index than they were a century ago. Since October 2000 when prices reached a 27-year low, cocoa has seen steady price increases as displayed in Figure 2. These increases in prices have been attributed to poor harvests, speculative activities and a general strong demand growth.

Seasonality plays an essential role in managing risk in agricultural commodities and it must be carefully analyzed when modeling cocoa spot and futures prices. The seasonal behavior of many commodity prices has been documented in numerous studies. Sorensen (2002) considers the evolution of agricultural commodity futures (corn, soybean, and wheat) by adding a deterministic seasonal price component to the two-factor model of Schwartz and Smith (2000). Geman and Nguyen (2005) model soybean prices through a deterministic seasonal component and two state variables representing the spot price and spot volatility, the latter being a proxy for inventory.

Choi and Longstaff (1985) note that there exists seasonality in most agricultural markets, the supply of goods being determined by harvesting cycles. As the degree of price uncertainty changes through the year, the volatility of commodity future prices shows a strong seasonal pattern. In the period prior to the harvest when the amount of crops is unknown, the price uncertainty is higher than after the harvest when crop yields are known to the market participants. Hence, the price volatility increases from the previous harvest to the information arrival on the new one, resulting in a seasonal pattern in volatility in addition to the price level seasonality. For instance, the severe drought in the former Soviet Union combined with floods in Canada during the year 2010 created a severe rise in wheat prices together with a high volatility as of the beginning of June, a long time before the harvest.

Many other factors influence the price of agricultural commodities, including natural disasters, political intervention like subsidies, and diseases. These factors ensure that agricultural prices rarely trade in a similar manner to one another or to other asset classes. Hence they provide to investors unique diversification benefits. In recent years, while supply has been constrained by a number of factors including land and water scarcity, demand has been driven further up by rapidly developing emerging markets as well as an increased consumption in developed countries.

Earlier studies have modeled seasonal trends in agricultural commodity prices by decomposing economic time series into their temporary and permanent components or detrending by filtering. Whereas the former ones are unable to capture the underlying data generation process, the latter are either one-sided filters or based on *ad hoc* procedures.

Historically, the majority of the work related to the modeling of seasonality in prices has concentrated on the seasonal pattern in the spot price; but this spot price is not observable for many commodities. Moreover, conclusions from such studies are mixed. Kaldor (1940) concludes that "supply curves of agricultural commodities are much less elastic and weather effects cause frequent and unpredictable shifts in the supply curve". In other words, producers are not able to rapidly adjust the supply level in response to price changes because of the time involved in planting new areas , while exogenous factors (for example, weather) do have random effects on supply levels. Another reason given by Fama and French (1987) is the high cost of storage due to their perishable nature. Other authors have pointed out that the dynamics of supply and demand result in a mean-reverting behavior of commodity prices (Brennan (1991), Gibson and Schwartz (1990).

In recent years, there has been a growing importance in obtaining reliable forward curves for commodities because of their price discovery virtues. Agricultural commodities are too perishable to be stored for long time periods or there may simply not be a cash-and-carry market at the time of trading. In such markets, commodity forward curves have become increasingly popular and important, in particular because of the key role of Futures contracts in hedging activities and the information conveyed in their prices. We can note that cocoa crop forecasters are known as 'pod counters' as they literally travel around the plantations in West Africa counting the number of cocoa pods in each tree to estimate output. Consequently, at any given date, there is a component of 'true expectation' of future spot prices present in the Future price since this one results from the confrontation of buy and sell orders for a given horizon date.

Hedging commodity price risk has been a standard activity for farmers and producers eager to protect their revenues by selling futures. In the early days of the Chicago Board of Trade (CBOT, 1848), the buyers of these futures used to be 'speculators' willing to take risk in order to make profits, and so providing liquidity to the hedgers. Today, they include hedge funds, pension funds and all actors wishing to be exposed to commodity prices, both for their diversification benefits and long-term returns due to the explosive growth of demand for food in a growing world population, as well as the biofuels sector. In such markets, the analysis of commodity forward curves has become increasingly important as the market players, hedgers (farmers and agrifood companies) and financial players consider the whole spectrum of liquid maturities available to them.

Seasonality is obviously one of the features which distinguish commodities from equity and fixed income markets. Its presence in the spot prices of agricultural and some energy commodities, naturally arising from seasonal patterns in supply (for example, harvest) and demand (for example, cold weather) is relatively well understood. It has been studied by a number of authors, for example, Milonas (1991) and is usually represented by a deterministic component. For the remaining part, and its possible representation by a factor model, the first state-variable has always been in the literature on the subject the commodity price. However, this spot price combines seasonal and stochastic features and may be unobservable. We will propose in this paper to use instead the average value of liquid contracts forward price, which is devoid of seasonality and conveys a more robust representation of the forward curve level. The second factor will be a quantity analogous to the stochastic convenience yield and account for the random changes in the shape of the forward curve while seasonality will be classically accounted for by a deterministic seasonal premium.

In the case of cocoa like for other commodities, the shape of the forward curve depicts anticipated prices and provides insight for hedging, storage and production decisions (Geman, 2005). Figure 4 shows a small sample of cocoa forward curves from the LIFFE.

On a number of observation dates, cocoa futures prices as observed on the LIFFE tend to be higher for deliveries in May, July and September (highest in July) than in March and December. This price trend is traced to the harvesting cycle of cocoa from the major producing countries as supply drives seasonality, like in most agricultural commodity markets, and the risk premium is highest in July when there is release of the information

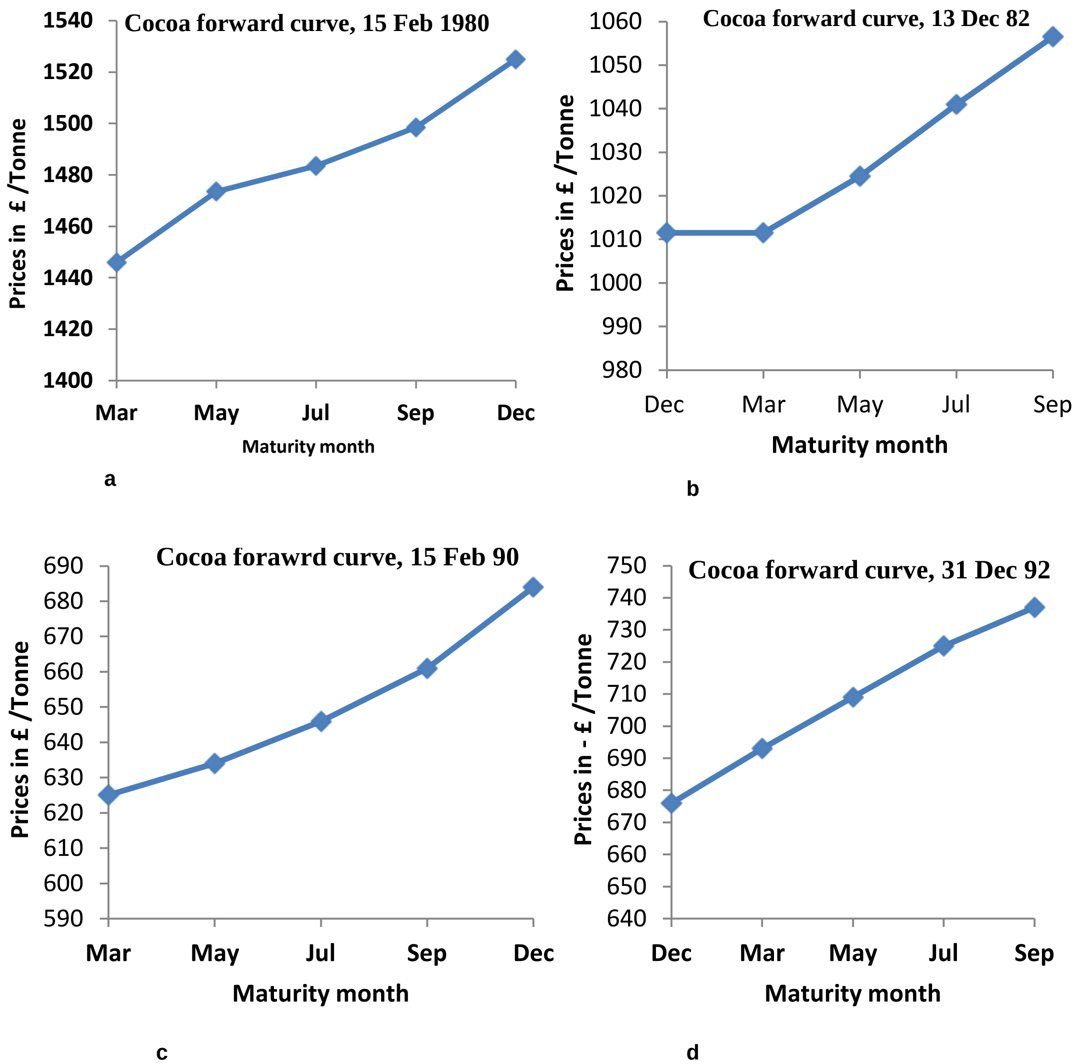


Figure 4. A sample of cocoa forward curves.

about the next harvest.

In general, we see strong seasonal price premia in those commodities which have strong seasonality in supply and/or demand. The price seasonality is more pronounced when inventories are low, since in these cases extreme shortage may occur just prior to a new harvest. However, in the case of cocoa there is only moderate seasonality in supply, because of the long harvest period of around 5 months. Compounding this, cocoa trees normally produce a smaller 'mid-crop' in between the main crops, accounting for around 20% of total production. In Table 1 we tabulate the main-crop months ('X') and mid-crop months ('o') for the main producing countries (UNCTAD, 2011). There are only 5 months of the year when main-crop harvesting is not occurring, and even then the mid-crop is being harvested. The combination of relatively high stocks and only weak seasonality explain why we find only mild seasonal premia in our later analysis.

As said before, the modeling framework aims at separating seasonal from stochastic features in cocoa forward curves. We follow Borovkova and Geman (2006) - hereafter BG - in not using the spot price as a state variable and introduce instead the average value of liquid forward contracts. Our framework is based on their approach and applied to cocoa futures data.

The price of any forward contract under the BG model is represented as the sum of an average forward price – meant to convey a global representation of the current forward curve, a deterministic seasonal component and a random component analogous to a stochastic convenience yield accounting for the random changes in the shape of the forward curve. Modeling the seasonal component as a deterministic variable is possible since

Table 1. Main-crop harvest months ('X') and smaller mid-crop harvest months ('o') for the main producing countries (UNCTAD, 2011).

Country	Proportion of worldwide production (%)	Calendar Month of Harvest											
		J	F	M	A	M	J	J	A	S	O	N	D
Ivory Coast	35	X	X	X		o	o	o	o		X	X	X
Ghana	19	X	X	X		o	o	o	o	X	X	X	X
Indonesia	14			o	o	o	o	o		X	X	X	X

the stochastic elements in seasonality will be embedded in the stochastic term. As we shall see, this representation significantly improves the well known cost-of-carry relationship.

Practical application exists in policy strategies meant to manage effects like price declines and volatilities in agriculture - dependent countries, many of which are among the least developed in the world.

MODEL DESCRIPTION

A factor model of the forward curve term structure allows one to capture stylized facts that are observed in Futures contracts of different maturities and at different moments in time in the most parsimonious and tractable form, namely through the dynamics of a small number of *state variables*. Term structure models were first developed in the fundamental case of interest rates, in order to account for the 'consistencies' between maturities implied by the no arbitrage assumption on the changes in spot rates. The same difficult problem holds for the term structure of commodity forward prices, with the further hurdle that such characteristics as seasonality and physical properties related to storability must be taken into account. For decades and today more than ever, has there been an acute interest in understanding and interpreting patterns of contango, backwardation, or kinks (existence of an inflection point in the curve) in commodity forward curves, from crude oil to gold or agricultural commodities.

The traditional theory of storage as originally proposed by Working (1932) provides a link between the term structure of futures prices and the level of inventories. This link, also known as "cost of carry relationship", establishes that in order to induce storage, futures prices have to be high enough over time to compensate inventory holders for the costs associated with storage. In a situation of contango, the futures price corresponds to the spot price increased by a positive storage cost. This explanation does not apply to explain the backwardation shape since the cost of storage is in nonnegative; hence, the necessity of introducing a *convenience yield* related to inventories and translating the benefit of owning the physical commodity readily available in the case of disruption in the production as opposed to a paper contract. The term convenience yield was introduced by the famous economist Kaldor (1939). It appeared in a finance setting in the spot- forward relationship proposed by Brennan and Schwartz (1985):

$$F(t,T) = S(t)e^{(r+c-y)(T-t)} \qquad (1)$$

whereby a futures contract at time t, for delivery at time T, is denoted by $F(t,T)$ and is a function of the current spot price S, foregone interest rate r and convenience yield y (which are both assumed constant[3] over time in this paper without any loss of generality), and cost of storage c. Note that c and r are positive; y is positive or negative depending on the period. The case $r + c - y < 0$ gives rise to backwardation and corresponds to a positive net convenience yield[4]. The contango shape occurs when $r + c - y$ is positive. Both shapes obviously reflect the situation at the date of analysis of supply, demand and inventory, as well as the anticipations of production and geopolitical factors in the future period, reflected in forward prices.

A look at historical data on daily quote for cocoa futures prices for the five nearby maturities in LIFFE shows the forward curve since 1980 to the late 2010 (except in a few instances) has generally been in backwardation, which means futures contracts further in maturity are priced higher (Figure 4 and A5 to A8 in the appendix).

In a general situation as considered in Gibson

[3] Note that all the results of the paper can easily be extended to stochastic interest rates following the methodology introduced in Geman (1989).

[4] As introduced by Kaldor (1939) and Working (1948), y accounts for the benefit of owning the physical commodity rather than a Futures contract written on it. The crucial value of this relationship is fully appreciated by the market participants. Defining the convenience yield y as net of cost of storage shows that the forward curve at date *t* is an increasing or decreasing function of the maturity T, depending on the sign of *(r − y)*, respectively called contango or backwardation.

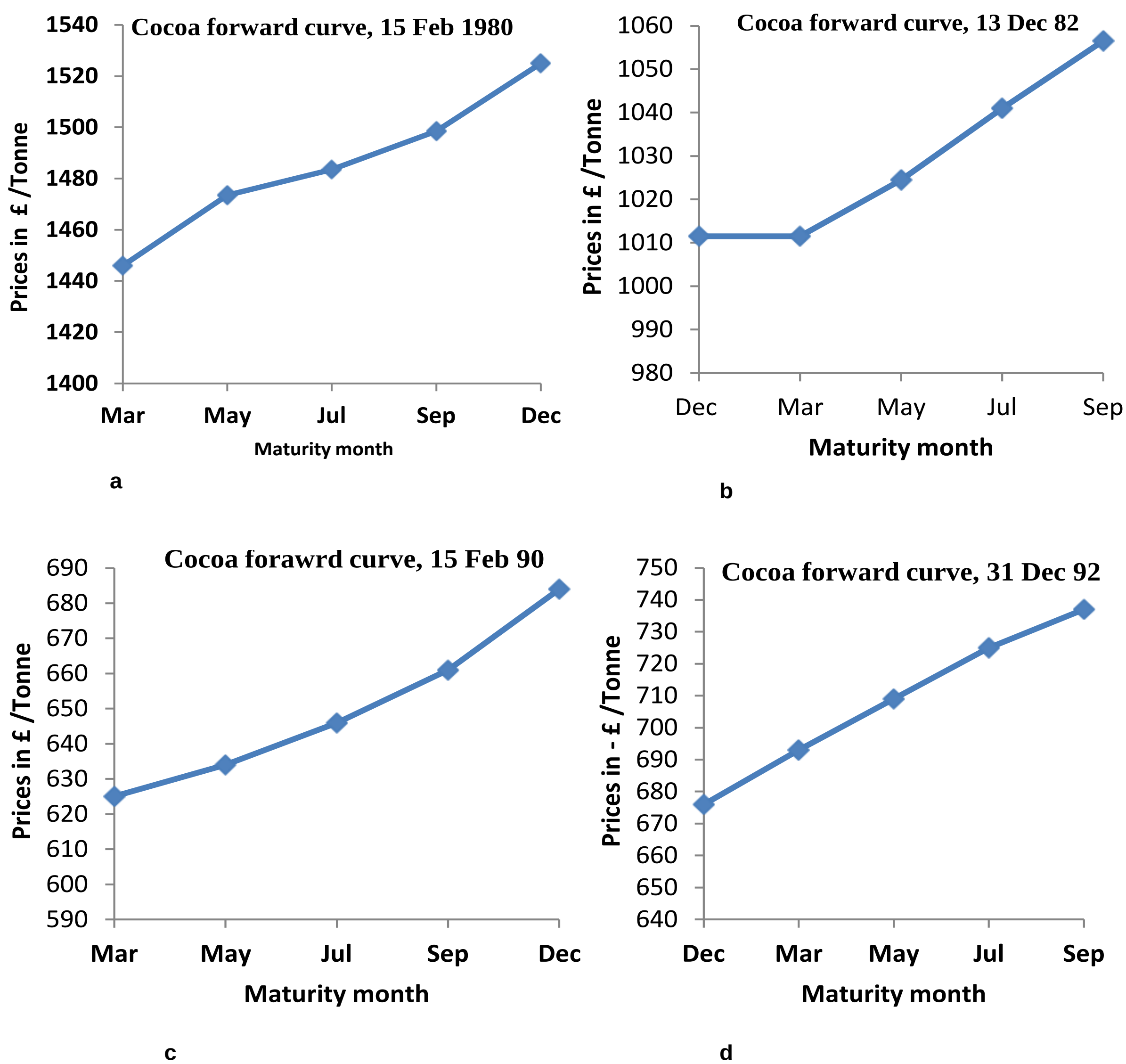


Figure 4. A sample of cocoa forward curves.

and Schwartz (1990), Schwartz (1997) and Litzenberger and Rabinowitz (1995), the convenience yield is defined as a stochastic process, *y(t)*. This conveys the fact that the "reward" received by the holder of the physical commodity changes with the world inventories and, in turn, economic agents' preferences for the physical good rather than a paper contract.

$$F(t,T) = S(t)e^{[r(t)-y(t)](T-t)} \quad (2)$$

Seasonality in the Futures price $F(t,T)$ lies in the cost of carry relationship and is embedded in the dependence on T of the convenience yield *y* related to the period (t,T). This is in sharp contrast with the case of a stock making a continuous dividend payment at the rate $g(t)$ as in Merton (1973), where the spot-forward relationship is given by;

$$F(t,T) = S(t)e^{[r(t)-g(t)](T-t)} \quad (3)$$

and where the rate $g(t)$ may vary with time but does not exhibit any dependence on the maturity T. We can rewrite equation (2) as:

$$F(t,T) = S(t)e^{[r(t)-y(t,T)](T-t)} \quad (4)$$

This expression reflects the character of time-spread option embedded in convenience yield, as discussed for instance in Heinkel et al. (1990).

Since defined by the intersection of futures buyers and sellers' quotes, a futures price conveys the expectation, conditionally to the available information at a date t, of the spot price at a future date. Futures prices of seasonal commodities are also driven by stochastic factors other than such extreme weather circumstances (outside the average seasonal pattern), political crises within producing countries, market risk aversion among the actors. We choose to express these external influences by a stochastic convenience yield, while directly incorporating a deterministic seasonal premium within the convenience yield.

Seasonal cost-of-carry model[5]

The first state variable is the average level of the forward curve, which we denote by $\overline{F}(t)$, defined as the geometric average of the current forward prices since we want to be in line with the geometric Brownian motion representation of prices in the classical finance literature- the assumption of geometric Brownian motion itself for spot or forward price being totally unnecessary in this paper.

$$\overline{F}(t) = \left(\prod_{T=1}^{N} F(t,T) \right)^{\frac{1}{N}} \tag{5}$$

where N is the most distant liquid maturity.

For the seasonal cost-of-carry model for any maturity T, we write

$$F(t,T) = \overline{F}(t) e^{s(T) - \gamma(t,T-t)(T-t)} \tag{6}$$

or equivalently,

$$\ln F(t,T) = \ln \overline{F}(t) + s(T) - \tau\gamma(t,\tau) \tag{7}$$

where $s(T)$ is the deterministic seasonal premium, and $\gamma(t,\tau)$, $\tau = T - t$, defined by the relationship above, is the second state variable, the *stochastic convenience yield net of seasonal premium*, observed at date t for time to maturity T.

Note that, in the seasonal cost-of-carry model above, we have separated the dependence on the maturity date T from the dependence on time to maturity $\tau = T - t$.

[5] Readers are referred to Borovkova-Geman (2006) for the use of this model in energy markets

The *maturity date* (calendar month in fact) influences the futures price via the seasonal premium s(T), while the time-to-maturity τ affects the futures price via the stochastic convenience yield $\gamma(t,\tau)$, aggregated up to maturity in the product $\tau\gamma(t,T)$. The average forward price $\overline{F}(t)$ only depends on the current date t of observation. The choice of this first state variable is a key feature of our approach.

Next, we specify the dynamics of the state variables. Defining $X(t) = \ln \overline{F}(t)$, we describe the dynamics of $X(t)$ and $\gamma(t)$ under the real probability measure P (no arguments of "risk-neutrality" are necessary in the developments below) by the stochastic differential equations

$$dX(t) = \alpha(\mu - X(t))dt + \sigma dW_1(t) \tag{8}$$

$$d\gamma^\tau(t) = -a^\tau \gamma^\tau(t)dt + \eta^\tau dW_2 \tag{9}$$

where σ, the volatility of $X(t)$ is assumed to be constant (but, brought together with volatilities of γ^τ, still leads to a term structure of Futures prices volatilities, as shown below in equation (8)). The convenience yields of all maturities $\gamma^\tau(t)$ are subject to a single source of uncertainty, represented by the Brownian motion W_2, uncorrelated with the Brownian motion W_1 driving the average forward price. We view γ^τ as fluctuating around zero over time, since shocks to inventories get eventually absorbed by adjustment of the production; $X(t)$ reverts to a long-term value μ, here assumed to be constant. The speeds of mean reversion are a^τ and α, respectively. The set (η^τ) $\tau = 1, 2, ..., N$ represents the stochastic convenience yield volatilities for different maturities. Note that the assumption of independence of W_1 and W_2 is totally reasonable given the way the state variables were defined.

Substituting the stochastic differential equations into equation (6), we can derive the dynamics of the Futures log-prices $Y(t, T) = \ln F(t,T)$ under the real probability measure:

$$dY(t,T) = \left[\alpha(\mu - X(t)) + \gamma^\tau(t)(a^\tau\tau + 1)\right]dt + \sigma dW_1(t) - \eta^\tau \tau dW_2(t) \tag{10}$$

So $Y(t,T)$ is obtained by integrating the above differential

Table 2. Parameters of the mean-reverting process $X(t)=\ln\overline{F}(t)$ for the period February 1980 to March 2009.

Parameter	Calibration result	Annualized value
α	0.001143	0.2857
μ	6.9367	6.9367
σ	0.014544	0.2300

equation with the initial condition $F(0,T)=\overline{F}(0)e^{s(T)-\gamma^{\tau}(0)(T-0)}$

Then $F(t,T)$ is log-normally distributed with variance

$$\delta^2(t,\tau)=\sigma^2+(\eta^{\tau}\tau)^2, \qquad \tau=T-t \tag{11}$$

APPLICATION TO COCOA FUTURES AND EMPIRICAL RESULTS

We obtained daily futures price data from LIFFE, in British Sterling Pounds per tonne, for the period 1980 to 2009, and calibrate using the technique as described in Borovkova and Geman (2006). In total 7583 days' of observations were used. We restricted our analysis to the first 5 contracts, comprising a futures curve of one year, since only the months of March, May, July, September and December trade on LIFFE.

Results of the calibration for the mean-reverting log-price process $X(t)=\ln\overline{F}(t)$, Equation (8), are presented in Table 2. The mean reversion speed *α* of 0.2857 indicates that the log-price (and hence the price) reverts to the mean in 1/0.2857 years, or in 3.5 years. This is a typical result for mean reversion in commodity prices Bessembinder et al (1995). The long term reversion value μ of 6.9367 for the log-price process corresponds to 1029 for the long-term reversion value of the price process itself. Note that the units of price are £ sterling as traded on LIFFE, compare with Figure 2(a) with cocoa prices in US dollars. Finally, the annualized volatility of 0.23 (23%) confirms cocoa as having moderate price volatility, typical of agricultural commodities. Note that the averaging effect of computing $\overline{F}(t)$ leads to a lower volatility value than would normally be calculated as the volatility of the spot price, using the first-to-mature futures price as a proxy. This is for two reasons. Firstly, the 'Samuelson effect' leads the first-to-mature futures price to be more volatile than later contracts, which $\overline{F}(t)$ includes. Secondly, since the daily changes to the empirical prices $F(t,T)$ are not perfectly correlated across all the contracts, the averaging procedure results in a lower overall volatility than the volatility of any one contract. This stability of $\overline{F}(t)$, and its explicit lack of seasonality, are two of the benefits of the BG (2006) model.

Figure 5a shows the seasonal premia for cocoa futures prices ranges (-0.00365 to 0.002868). The seasonal premia in coffee futures are supplied for comparison purposes in Figure 5b.

In the case of cocoa, the seasonal premia $s(T)$ are very small; they represent the proportion by which a given month exceeds the average forward price $\overline{F}$. As expected, the seasonal premium is negative in December and March, which represents the major harvesting period for cocoa in West Africa. Futures expiring in May, July or September are at premium with respect to the average futures price level. This is also in line with the mid-crop or lean harvest when demand for cocoa is expected to outstrip supply and stocks are being depleted. Based on these small but varying premia, we can infer (as is the case) that the main harvest occurs between September and December when the premium goes from positive to negative. The premium is highest in July when inventories are at their lowest and uncertainty about the next main crop is maximal (the time called 'weather period' in the wheat market, indicating the importance of any weather news). In September, the harvest has started and more precise levels of crop outlook are available, hence a reduction in the uncertainty risk premium, although some uncertainty still remains since the remainder of the harvest period could still experience disruption.

In the case of coffee, we see higher seasonal premia. Coffee together with cocoa and tea form what are generally termed 'the tropical beverages'. Both cocoa and coffee are produced in the tropics. Coffee, however, is harvested in only specific months of the year, with a clear harvest and no-harvest period. This contributes to the greater seasonal premia. The values in Figure 5(b) implies a mean premium of 1.5% for September coffee over the mean, and a May discount of 1%. These are still low compared to the extreme seasonality noted in BG (2006) for the case of natural gas, in which December expiry gas futures exceeded the mean by almost 30%.

We obtain the estimates for the stochastic convenience yield series for all available times to maturity from the residuals of equation (10) as:

$$\gamma(t,\tau)=\frac{1}{\tau}[\ln\overline{F}(t)-lnF(t,T)+s(T)]$$

and display these in Table 3. The values of $a^{\tau}a$, the mean reversion speeds for the various maturities, indicate rapid reversion in $\frac{1}{a^{\tau}}$ years, with the annualized values corresponding to reversion speeds between 0.174 and 0.237 years, that is, the stochastic convenience yield - net of seasonal premium - reverts rapidly to 0. Thus we

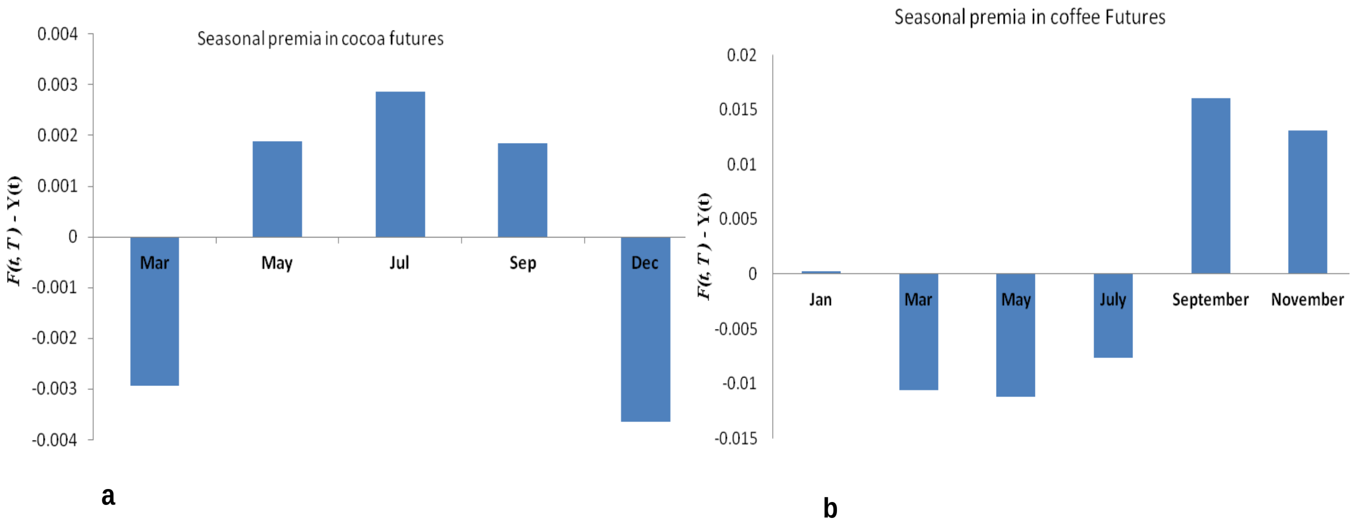


Figure 5. (a) The set of Seasonal Premia $s(T)$ obtained for cocoa; (b), the set of seasonal premia $s(T)$ obtained for coffee analyzed over an identical study period.

see that the overall premium of a given log-futures price $F(t,T)$ over the average value $\overline{F}(t)$, while stochastic, reverts rapidly to that calibrated as the fixed seasonal premium $s(T)$, lending credence to the utility of the BG (2006) model.

Figure 6a depicts the version of the "Samuelson effect" corresponding to our setting, namely the decreasing volatility of the stochastic convenience yield with the time to maturity of the future contract, as observed by BG (2006) for natural gas and electricity. For the sake of comparison and in order to exhibit another instance of this property in the class of 'softs', we plot in Figure 6b the term structure of the convenience yield volatilities in the case of coffee. These results tells us that convenience yield varies more rapidly as we approach maturity, just as price itself varies more rapidly in the 'original' Samuelson effect, which refers to the volatility of price.

Conclusion

Agricultural commodities represent a fascinating asset class because of their indispensable place in human life and from a theoretical standpoint. Having a better understanding of the evolution over time of the forward curve is crucial for farmers and agrifood companies who wish to hedge their revenues across the year and investors who wish to get exposure to the commodity markets.

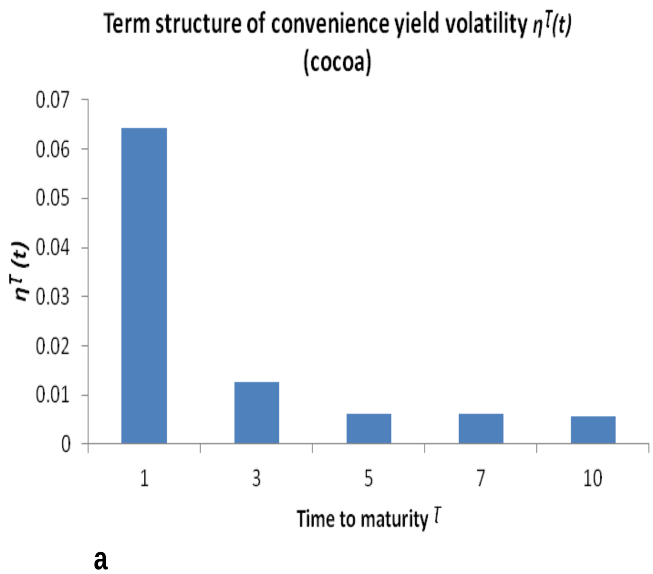
In the case of cocoa, a food crop which is consumed in various forms around the world, we have shown that a two- state variable model which avoids the opaque spot price, allows one, together with seasonality, to calibrate in a very satisfactory manner the data for the period extending from 1980 to 2009. Prices of food crops typically follow a seasonal pattern, falling immediately after the harvest and rising thereafter until the next harvest, as farmers and merchants store some supplies to meet consumer demand throughout the year. Cocoa however has a very mild seasonal price trend because it is harvested over the entire year (varying only in harvest quantities between major and minor harvests).

The knowledge of the seasonal pattern of commodity prices is essential to effective potential price stabilization policies on the world market. For economies that depend on cocoa export for foreign revenue, knowledge of the seasonal pattern of cocoa price and its magnitude is essential to the formulation of effective policies for stabilizing foreign revenue and the income of farmers. The results of our analysis provide a meaningful start point for cocoa price policy. Although cocoa prices currently display only weak seasonality, we expect the seasonality would rise if the stocks to grindings ratio were to fall substantially. However, despite most production being concentrated in West Africa, the length of the harvest period, and the presence of the inter-harvest mid-crop secondary harvest, means that cocoa will never display the strong seasonality witnessed currently in the case of corn or wheat.

The cocoa market is subject to a moderate degree of volatility due to global cocoa production being limited to just eight countries (that is, eight countries serving the global demand). This presents attractive hedging and trading opportunities for cocoa traders around the world. The results of our analysis provide traders one less factor to worry about as seasonal trend in cocoa price is empirically shown to be insignificant.

Table 3. Gamma parameters for the period February 1980 to March 2009.

Contract maturity	Calibrated values			
	a , daily	*a,* annualized	η	η, annualized
March	0.018051	4.513	0.022871	0.361
May	0.022197	5.549	0.015536	0.245
July	0.022976	5.744	0.01136	0.179
September	0.017917	4.479	0.008268	0.131
December	0.016854	4.213	0.006503	0.103



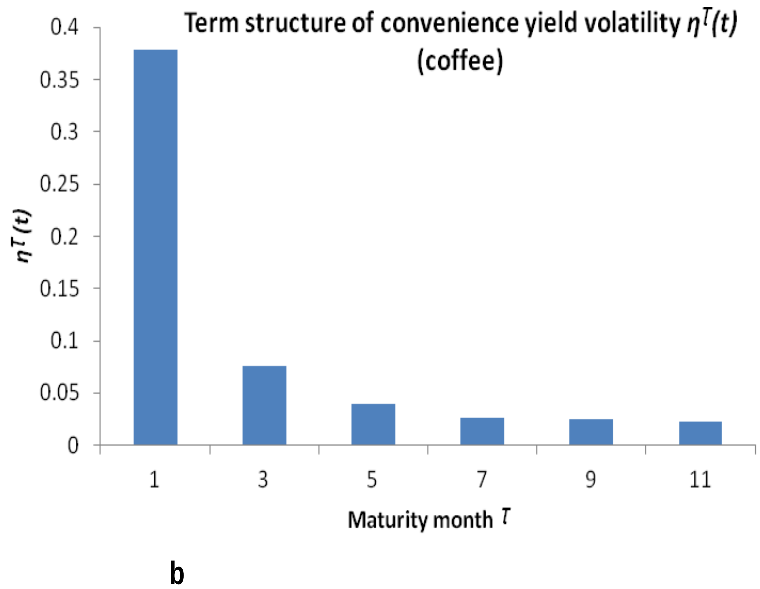


Figure 6. (a) Term structure of stochastic convenience yield volatility for cocoa; (b), term structure of stochastic convenience yield volatility for coffee.

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Appendix

We display below a sample of cocoa forward curves, observed at different dates in the year and across the period of observation.

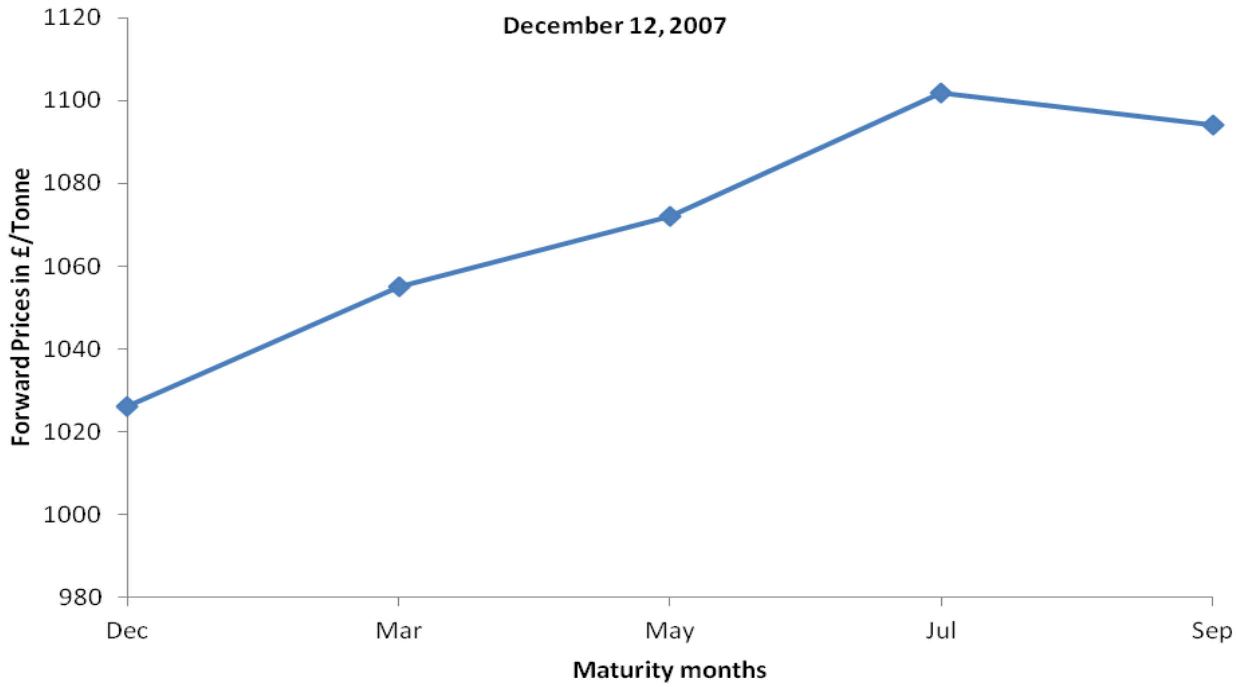

Figure A1. Cocoa forward curve; December 12, 2007.

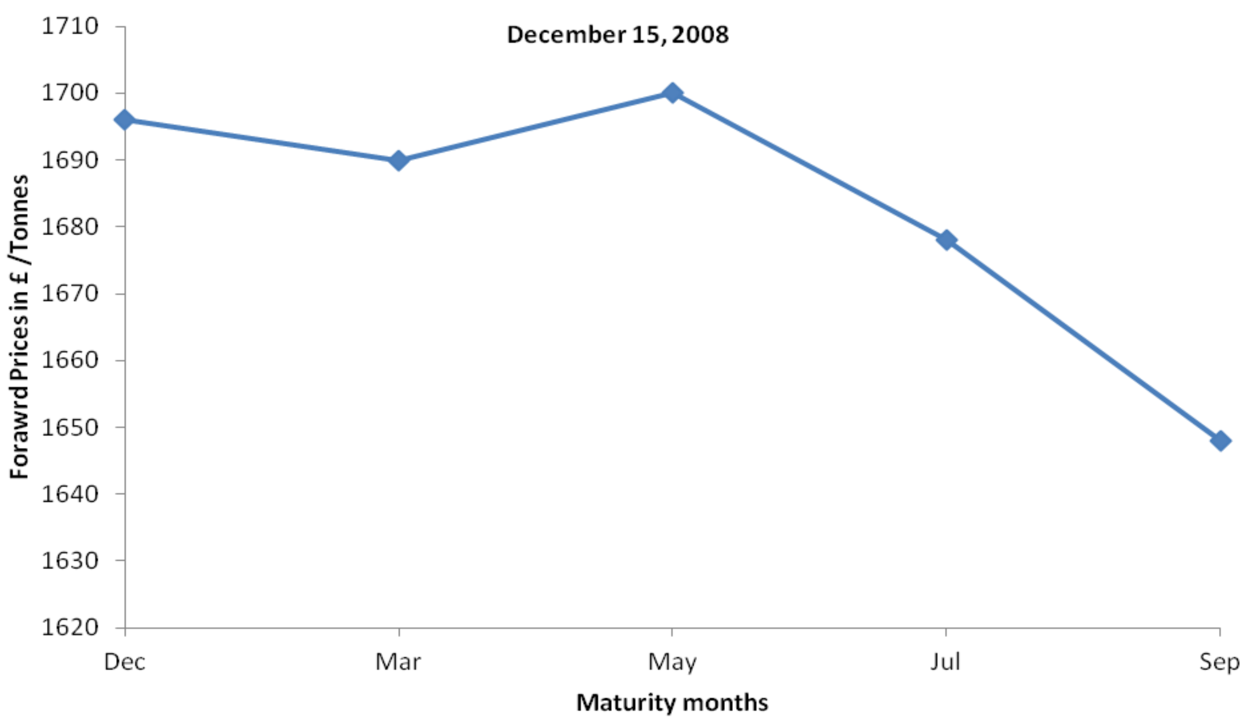

Figure A2. cocoa forward curve; December 15, 2008.

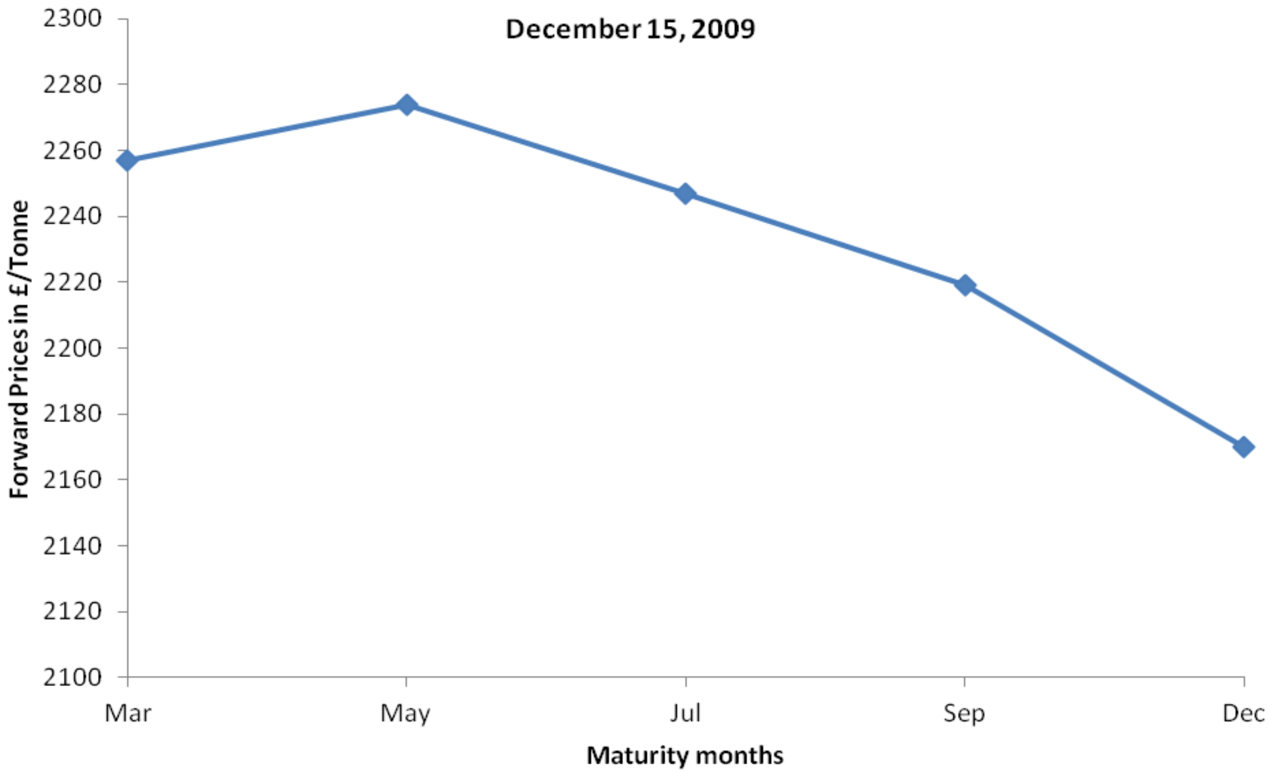

Figure A3. Cocoa forward curve; December 15, 2009.

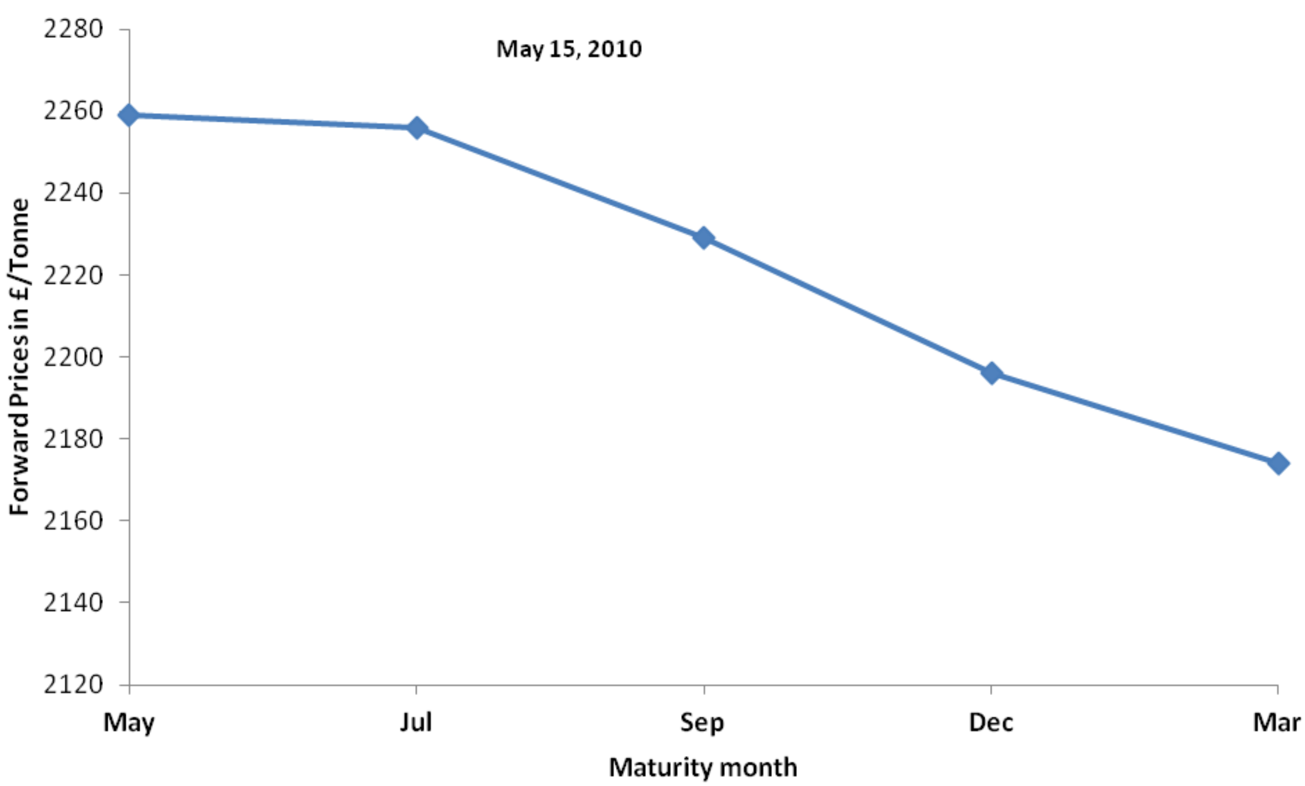

Figure A4. Cocoa forward curve; May 15, 2010.

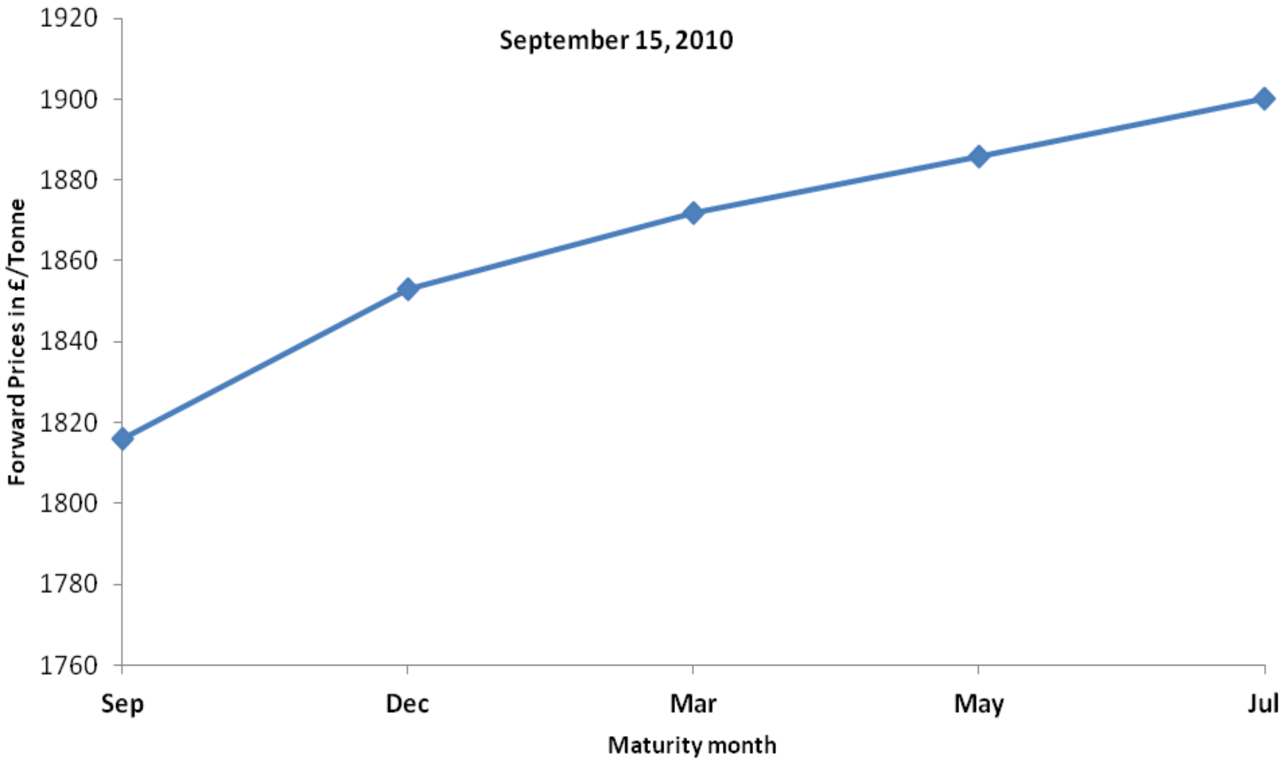

Figure A5. Cocoa forward curve; September 15, 2010.

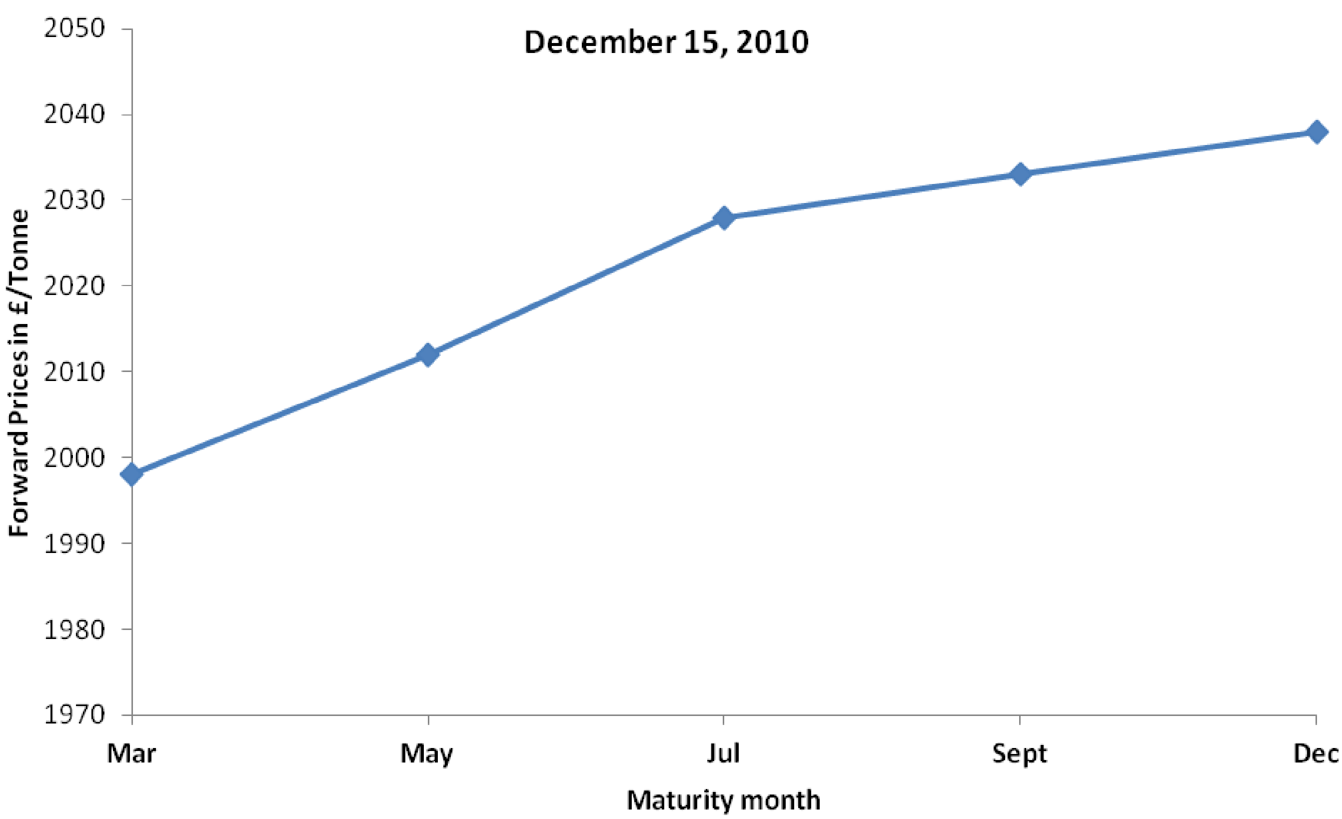

Figure A6. Cocoa forward curve; December 15, 2010.

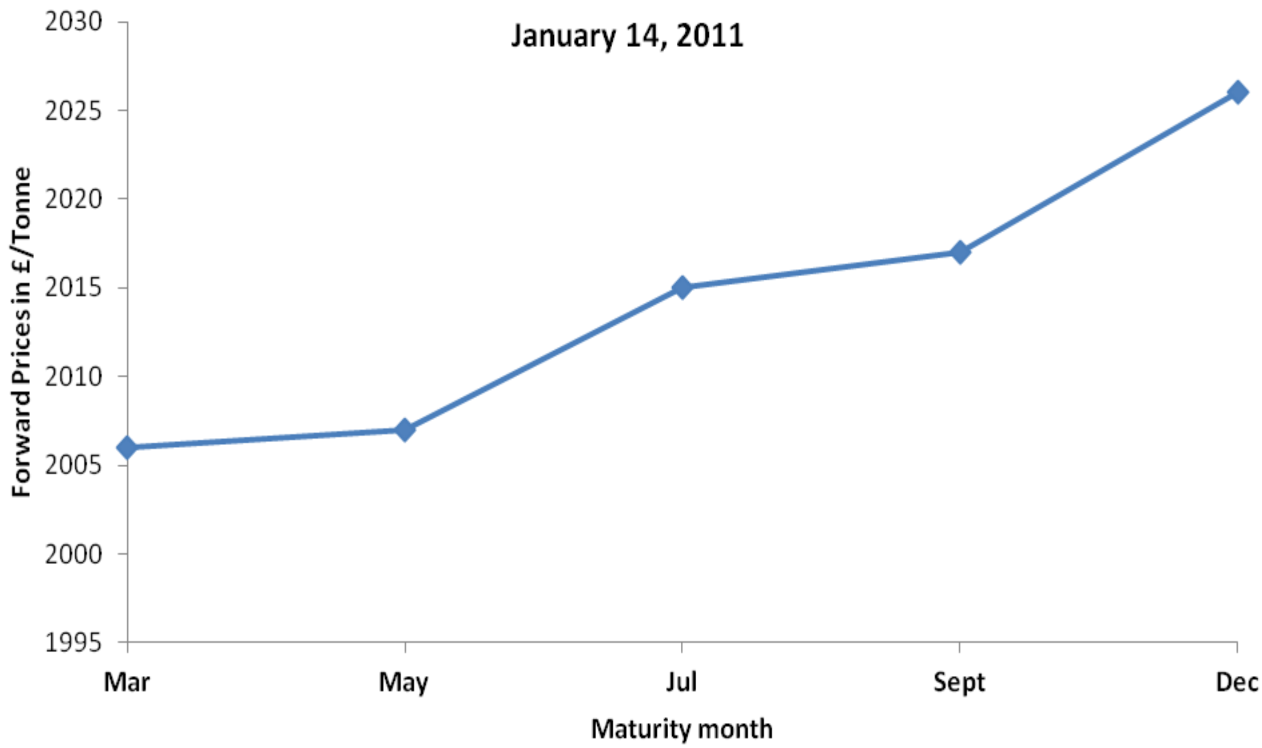

Figure A7. Cocoa forward curve; January 14, 2011.

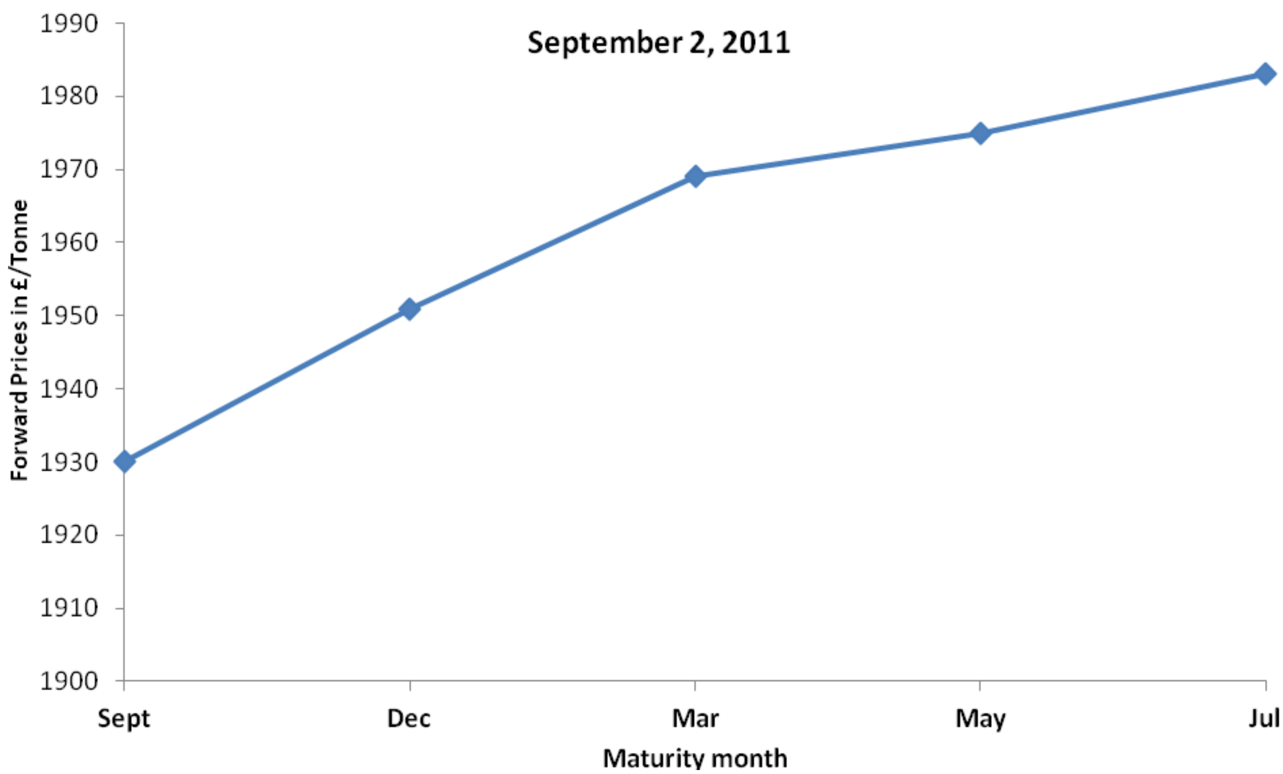

Figure A8. Cocoa forward curve; September 2, 2011.