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# On the Lease Rate, Convenience Yield and Speculative Effects in the Gold Futures Market

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## Abstract

By examining data on the gold forward offered rate (GOFO) and lease rates over the period 1996- 2009, we conclude that the convenience yield of gold is better approximated by the lease rate than the interest-adjusted spread of Fama & French (1983). Using the latter quantity, we study the relationship between gold leasing and the level of COMEX discretionary inventory and exhibit that lease rates are negatively related to inventories. We also show that Futures prices have increasingly exceeded forward prices over the period, and this effect increases with the speculative pressure and the maturity of the contracts.

Key words: gold futures market, convenience yield, gold lease rate, speculative pressure

JEL Classification: C22, E44, G15

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We study in this paper the effects of the gold lease rate, speculative and pricing pressure in the gold market. The gold market of today is a much different market than it was 10 years ago. The New York Commodity Exchange (COMEX) is witnessing historically low lease rates, decreasing hedging activity and steadily rising non-commercial open interest. For a market that was once dominated by commercial<sup>1</sup> positions, this is a fundamental change in the *status quo*. This ever-increasing percentage of non-commercial open interest results from the intense activity of two new classes of gold investors: gold exchange traded funds (ETF) and non-commercial speculators. With their recent proliferation and rising influence, ETFs are playing an ever more prominent and escalating role in the complex dynamics that now affect the price of gold. Speculators are also betting on the gold price in increasing numbers as commodities are now common in investment portfolios and major currencies become demonetised. The gold market, once the domain of an eclectic group colloquially known as "gold bugs", is now readily accessible to any investor wishing to put savings into gold bullion. However, historically, central banks and bullion banks were the primary gold trading agents. Presently, with gold investment demand increasing and trades facilitated by electronic trading platforms, non-traditional investors are seeking to direct their capital to gold. More generally, gold is increasingly becoming a financial investment rather than a commodity in the traditional sense.

Before the recent increase in investors' interest and the creation of ETFs, central banks and bullion banks were among the largest operators in the gold futures market. Indeed, prior to the Washington Agreement on Gold sales in 1999, central banks bought and sold significant quantities of gold, thereby either directly or indirectly affecting the price of bullion. This idea that central bank sales could impact the price of gold was examined by Salant and Henderson (1978). They found that a government gold auction served to depress the price of gold; yet, under some circumstances, it could also pressure the price of gold to rise<sup>2</sup> at a rate that exceeded the real rate of interest. These results lead Salant and Henderson to suggest that the Hotelling (1931) model of exhaustible resources is unable to properly describe gold price movements. More specifically, it is not capable of predicting the observed increases in the gold price that occur at rates exceeding the prevailing rate of interest. Their results were related to a period prior to 1978, contrasting with the remarkably steady increase of gold prices over the past decade, as depicted in figure (1) below. Moreover, the recent purchase by the Central Bank of India of 200 million ounces of gold from the IMF, followed by Mauritius and Sri Lanka, show that central banks are more concerned than ever by their gold reserves.

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<sup>1</sup> Held by hedgers

<sup>2</sup> In percentage terms.

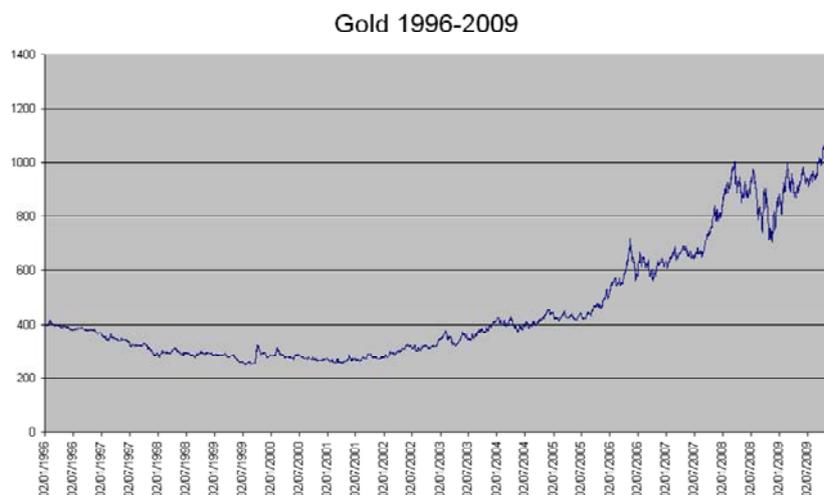


FIGURE 1: EVOLUTION OF THE GOLD PRICE OVER THE PERIOD SPANNING FROM 1996 UNTIL 2009.

## I. On Speculative Pressure and its Possible Effects

The gold futures market has undergone significant changes in the last two decades. In recent times, the market has been dominated by non-commercial long players and has also witnessed positive values of the speculative pressure for an extended period of time. This is a significant break from the historical precedent of high levels of hedging activity.

Keynes' theory of normal backwardation implies that the net supply of commercial futures contracts, or hedging pressure, affects the equilibrium futures prices. This can lead to rises or falls in futures prices over time. Such an equilibrium trend can be interpreted as the risk-premium. When hedgers take long positions, the equilibrium achieved is such that futures prices tend to decrease over time. Conversely, in a market where short hedging dominates, futures prices tend to increase over time. However, not all market agents can freely diversify their portfolios and, as a result, a price bias is created. In a Keynesian world, producers will hedge their short positions by taking a corresponding long futures position in the physical commodity. The resulting net supply of futures contracts, termed the hedging pressure, creates a downward pressure on futures prices of distant maturities. This decrease, called backwardation is often termed “normal backwardation” as it was the shape usually observed in commodities during the previous decades.

Speculators are economic agents that usually acquire long positions in futures markets. In order to enter into a long futures position, they require a premium as compensation for the risk that they bear, the aforementioned risk-premium. However, speculators are not the only agents that take these long positions. Producers and refiners facing inventory problems may

also take long futures positions. Combined, both producers' and speculators' long positions can result in an upward bias, or contango, in futures prices. This situation can be particularly acute when inventories are more variable than prices.

In support of the hedging pressure hypothesis, Bessembinder (1992) found that returns in the agricultural commodity markets vary with the net holdings of hedgers. Using cross-sectional commodity data along with trader position data, Bessembinder showed that hedging pressure has an explanatory power for risk premia. More specifically, he found that unconditional futures returns have mean returns that do not differ significantly from zero. In contrast, when those returns were conditioned on hedging pressure, he reported that mean futures returns were significantly different than zero.

In more recent work on hedging effects, de Roon, Nijman and Veld (2000) modelled the returns on a portfolio consisting of non-marketable risks, investment assets and futures contracts. Such a model allowed for the existence of a relationship between the risk-premium and the hedging pressure, which was calculated using bi-monthly observations of traders' positions taken from the CFTC. Their model was such that hedging pressure arises from risks that agents either do not, or cannot, hedge as a result of market frictions or transaction costs. Under such assumptions, de Roon et al. showed that hedging pressure leads to price bias. They also found that returns were influenced by hedging pressure, not just the hedging pressure in the commodity's own market but across commodity markets. By controlling for price pressure, measured as a change in hedging pressure, de Roon et al. showed that hedging pressure is responsible for the formation of price bias in futures prices. Additionally, they found that hedging pressure influences not just futures returns, but returns on the underlying asset of the futures contract.

To understand how speculative agents can affect the gold futures market, we examined the open interest data from the Commodity Futures Trading Commission (CFTC) Commitment of Traders (CoT) report. The CoT report contains open interest data regarding the various trading positions of the futures market for gold<sup>3</sup>. The open interest data is separated into various categories that represent the possible trading positions. These are subsequently decomposed into various reporting categories themselves. The primary distinction is between reportable and non-reportable positions. Reportable positions can be further partitioned into commercial and non-commercial positions. According to CFTC regulations, commercial positions consist of those market positions used primarily for hedging<sup>4</sup>. We thus identify commercial open interest with hedging activity. Conversely, non-commercial positions are

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<sup>3</sup> In the CFTC CoT report, there are multiple categories of data. We are interested in commercial and non-commercial (all) data. The category denoted by "other" is not utilized in this paper but contains information regarding the remaining futures contracts. In short, this category contains traders not categorized as either "long" or "short" under the CFTC reporting framework. Specifically, we use non-commercial long, short and spread (all), commercial long and short (all) and non-reporting long and short (all). We also use the data for total open interest (all).

<sup>4</sup> We refer to the definition of a "hedging transaction" as specified in the CFTC Electronic Code of Federal Regulations (e-CFR) section 1221(b)(2)(A). This definition details a "hedging transaction" to be "any transaction that a taxpayer enters into in the normal course of the taxpayer's trade or business primarily for various risk management activities."

identified with speculative activity. This category includes positions taken by speculative institutions like hedge funds, for example. The classification is not rigorous, however, as some commercial positions may be speculative in nature while some non-commercial positions may be associated with hedging activity<sup>5</sup>. This classification can be further subdivided into long, short and, in the case of the non-commercial data, spread positions. These data comprise a subset of the total open interest. Since aggregate activity by hedge funds and other institutions are likely to increase the speculative activity, this may result in a net positive number of long futures contracts in comparison to short futures contracts. Subsequently, it is possible to calculate a measure of the excess of long contracts over short contracts.

To quantify the speculative activity in the gold futures market we define the speculative pressure as the analogue to the hedging pressure of de Roon et al (2000). The speculative pressure,  $\psi_t$ , is given by equation (1):

$$\psi_t = \frac{NC_t^{long,all} - NC_t^{short,all}}{NC_t^{long,all} + NC_t^{short,all} + 2NC_t^{spread,all}} \quad (1)$$

where  $NC$  indicates non-commercial open interest. Speculative pressure can be a measure for characterizing a commodity market as either net long speculation or net short hedging if the open interest of reporting speculative long open interest<sup>6</sup> exceeds (or is less than) speculative short open interest. This interpretation is backed by the work of Hirshleifer (1990) who identifies hedging pressure with the supply of futures contracts. Hirshleifer states that high values of hedging pressure result in a decrease in futures prices in comparison to the expected future spot price. This effect manifests itself through the existence of a downward bias that is present in futures prices.

Our inventory data consists of daily observations of gold bullion inventories held by the COMEX market making members and is recorded in troy ounces. These aggregated inventory data are subdivided into two categories: registered stocks and eligible stocks. The "registered" designation implies that the bullion is eligible for delivery against a futures contract. Conversely, the "eligible" designation refers to bullion inventories kept in the warehouse but not yet certified for delivery. In general, these inventory levels are not necessarily indicative of underlying supply and demand conditions. This is because the major depository institutions<sup>7</sup> choose independently when to deposit their stocks. Additionally, other bullion warehouses exist for which COMEX does not possess inventory statistics. These observations span the period from the beginning of January 1996 until the end of October 2009 and were used to construct a weekly series of de-trended and

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<sup>5</sup> For example, we know that banks sometimes hedge because they have an OTC contract with a customer who may be hedging or speculating. Consequently, this may blur the distinction between hedgers and speculators and results in noisy speculator (i.e., non-commercial) data. But by in large, the classification holds.

<sup>6</sup> i.e., non-commercial long open interest

<sup>7</sup> These include Brink's Inc., Scotia Mocatta, HSBC Bank, USA and Manfra, Tordella & Brookes, Inc.

studentized inventory levels similar to the procedure described in Dincerler et al. (2005). Initially, we created an inventory innovation series using an ARIMA(0,1,0) model and then divided the residuals by their standard deviation the resulting series of innovations. This procedure resulted in a series of discretionary inventories, as described by Routledge, Seppi and Spatt (2000). The value of using discretionary inventory levels lies with the observation that this measure of inventory is indicative of the balance between the current and future consumption value of the commodity. The level of discretionary inventory is therefore closely associated with the concept of a convenience yield.

Our open interest data consists of weekly open interest values for commercial, non-commercial and small trader positions collected from the CFTC Commitment of Traders reports. The data span the period from January 1996 until October 2009 and consist of a total of 712 weekly observations.

From daily observations of the gold futures price, we compiled a weekly series of futures prices for the first, third, sixth and twelfth nearby gold futures contracts traded on COMEX that resulted in a series for which prices are observed every Tuesday. In order to construct a continuous returns series, we used the method outlined in de Roon et al (2000). This series of Tuesday prices corresponds to the dates on which the Commodity Futures Trading Commission (CFTC) releases their weekly Commitment of Traders (CoT) report containing the levels of market open interest.

## II. The Effect of Speculative Pressure on Returns

In recent years, commodities have become a very popular investment and are now present in the portfolios of investors as an asset class that provides not only diversification but profitable returns. To that effect, it is apropos to study the factors affecting commodity futures returns. We do so using a regression model similar to that of de Roon et al. (2000):

$$r_{i,t} = \alpha + \beta_1 r_{i,t-1} + \beta_2 r_t^{S\&P500} + \beta_3 \psi_t + \varepsilon_t \quad (2)$$

Equation (2) regresses the return of the  $i^{th}$  returns series for the 1, 3, 6 and 12 month futures contracts on past returns,  $r_{i,t-1}$ , the return on the S&P 500 market portfolio,  $r_t^{S\&P500}$ , and the speculative pressure,  $\psi_t$ . Least-squares coefficient estimates and their associated standard errors are reported in table (I).

[Table I about here]

For all returns series, the constant coefficient is small, positive and not statistically significant. A similar result holds for the coefficient of lagged returns, confirming the classical result that past returns are not reliable predictors of current futures returns. The returns on gold futures contracts are positively related to the returns on the market portfolio,

but the result is not statistically significant. We note in particular that speculative pressure exerts a positive influence on returns. For all months, the speculative pressure coefficient is positive and highly significant, having a consistent value of approximately 0.008. As a robustness check, we include the price pressure discussed in de Roon et al (2000) that results from increased demand for contracts and takes the form of a change in speculative pressure. Specifically, an increase in demand for futures contracts will result in a temporary upward futures price bias. This is the price pressure hypothesis. In order to compare the regression coefficients on speculative pressure and price pressure, we divide each term by its respective standard deviation. This leads to the regression model described in equation (2):

$$r_{i,t} = \alpha + \beta_1 r_{i,t-1} + \beta_2 r_t^{SNP500} + \beta_3 \frac{\psi_t}{\sigma(\psi_t)} + \beta_4 \frac{\Delta\psi_t}{\sigma(\Delta\psi_t)} + \varepsilon_t \quad (2)$$

The associated coefficient estimates are shown in Table (II).

[Table II about here]

We confirm that neither the constant term nor past returns influence current returns, which is consistent with our previous findings. Moreover, there seems to be a more pronounced speculative effect for the longer maturity futures returns series, which is consistent with the fact that a number of speculators make bets on the long-term price of gold (also expressed by the purchase of shares of gold mining companies). Despite this, price pressure effects dominate speculative pressure effects, the former being highly significant while the latter exhibits exceedingly small  $p$ -values. Thus, for long-term returns series, there is a conclusive effect that speculative pressure, in part, determines futures returns even after controlling for past returns. Additionally, we note that the price pressure coefficient,  $\beta_4$ , is positive and highly significant across all returns series, suggesting that demand for futures contracts, in part, determines futures risk premia.

### III. Central Banks and the Gold Leasing Market

For most of its existence, the gold futures market was used for the gold carry trade<sup>8</sup>. The carry trade was facilitated by central banks wishing to earn a return on their bullion inventories. This trade involved borrowing gold from the inventories of central banks and consisted of either leasing or swapping gold in exchange for a fee. In this manner, the gold could be leased at a relatively low rate from the central bank and then sold quickly on the spot market. The proceeds from such a sale could then be invested at the London Interbank Offer Rate (LIBOR) or in Treasury bills. Because the lease rates charged by the central banks were less than the LIBOR rate, this was on average a profitable trading strategy as long as the spot price did not move significantly. The leasing institution was essentially able to earn the Gold Forward Offered (GOFO) rate as a return. However, since late 2001, the

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<sup>8</sup> See "Bullish on Bullion" by Peter Madigan, Risk, February 1, 2008.

profitability of the short-maturity carry trade has diminished. Rising gold prices have increased the risk and diminished the trade's profitability due to an accompanying increase in repayment costs<sup>9</sup>.

To gain insight into this aspect of the gold market, we will consider how a central bank can proactively use gold to create a yield from its bullion inventories. A central bank can lease gold to the market using two methods. The first is by simply leasing bullion to another institution. The second method is essentially a swap whereby gold is exchanged for U.S. dollars. The leasing of gold by central banks is relatively straightforward as a transaction. The rate at which gold is leased is derived from the difference between the LIBOR and GOFO rates and is given by equation (3):

$$\textit{lease} = \textit{LIBOR} - \textit{GOFO} \quad (3)$$

This is considered a derived rate since it is not set independently, but rather arises from the difference between the market quoted LIBOR and GOFO rates. A leasing transaction would involve a central bank transferring ownership of the gold to the leasing institution at a rate defined by equation (3), for the duration of the loan. At a later date, the leasing institution, or borrower, would buy back or simply return the gold and pay the central bank the original loan plus the lease rate as interest. Hence, the leasing bank is able to generate a profit using this transaction while the central bank earns the difference between the LIBOR and GOFO on its idle gold inventories.

The second type of transaction is actually less a swap than a sale and repurchase agreement. In a gold swap, a central bank is willing to exchange its gold bullion for cash. With the leasing transaction, the ownership title of the gold is transferred to the leasing institution through an actual sale, with the added condition that the central bank agrees to repurchase the gold from the borrower at some forward date. In selling gold to the borrower, the central bank receives U.S. dollars and, in addition to agreeing to a repurchase, the central bank also pays interest to the leasing institution at the GOFO rate. The GOFO is thus the interest rate charged on a loan denominated in dollars and using gold as collateral. Once in his/her possession, the borrower is free to sell the gold on the spot market and invest the proceeds as before, or to simply hold the bullion and earn the secured GOFO rate.

In a gold swap, the central bank pays GOFO to the borrower. Since the GOFO rate is less than the LIBOR rate, the advantage for the central bank is that it exchanges gold for cash reserves that can be invested at a rate that exceeds the GOFO. The difference between the investment at LIBOR and the payment of GOFO is the lease rate.

Unlike the derived lease rate, the GOFO rate is the mean of a series of rates offered by the market making members of the London Bullion Market Association (LBMA). Hence, these market makers can, to some extent, adjust the GOFO up or down depending on market conditions. Thus, we can think of the GOFO rate as an observable signal from the

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<sup>9</sup> The gold carry trade (like all carry trades) is profitable when gold prices are either stable or decreasing.

participating banks. When the GOFO rates are set such that there is a high differential with the LIBOR, the market makers are interested in exchanging bullion for U.S. dollars, a signal that banks are interested in obtaining cash using gold as collateral. This usually occurs under conditions of high demand for bullion. When the GOFO rate is set close to the LIBOR, neither strategy explained previously is highly profitable and consequently, the banks signal a willingness to contribute liquidity. This situation occurs under low demand for gold, but also when the gold price is high and/or rising. In light of this, we can think of the lease rate as a proxy for aggregate demand for physical gold or, more accurately, as the convenience yield of gold (see Kaldor (1939)).

#### IV. Convenience Yield and Lease Rate

We explained that in a gold leasing transaction, the central bank earns the difference between the prevailing LIBOR and current GOFO rate. As a result, the lease rate is effectively an observable quantity related to the convenience yield, the latter being itself obtained from the interest-adjusted basis. Following the “basis” introduced by Telser (1958), Fama and French (1988) define the interest-adjusted basis by

$$IAB_t = \frac{f(t,T) - S(t)}{S(t)} - R(t,T) \quad (4)$$

This is the difference between the basis  $(f(t,T) - S(t))/S(t)$  and the interest rate. Moreover, the spot-forward relationship can be written as

$$\frac{f(t,T) - S(t)}{S(t)} = R(t,T) + w(t,T) - c(t,T) \quad (5)$$

where  $f(t,T)$  is the forward price,  $S(t)$  is the spot price at time  $t$ ,  $R(t,T)$  is the interest rate over the period from  $t$  to  $T$ , and  $w(t,T)$  and  $c(t,T)$  are the relative warehousing cost and relative convenience yield, respectively. Together, equations (4) and (5) provide us with the following relationship

$$IAB_t = w(t,T) - c(t,T) \quad (6)$$

#### V. Price Differences between Forward and Futures Contracts

In a seminal paper, Cox, Ingersoll and Ross (1981) have exhibited that even in absence of credit risk, forward and futures prices are not equal under stochastic interest rates. French (1983) and Park and Chen (1985) empirically confirmed this result and exhibited that, on average, futures prices exceeded forward prices. If we consider a gold forward contract of maturity  $T$ , we have at date 0 the following equality

$$f_0 = S_0 e^{(r+w-c)T} \quad (7)$$

where  $r$  is the cost of financing over the period  $(0, T)$ . To express equation (7) in terms relevant to the gold market, we recall that a central bank can either lease the gold directly, or engage in a gold swap with a suitable counterparty. In the former case, the central bank earns the lease rate as a pure result of holding physical gold. Since this quantity is earned by holding a physical inventory, the lease rate can be interpreted as the convenience yield of gold. The counterparty to the leasing agreement is charged the lease rate on the bullion loan, but can immediately sell the gold on the spot market and invest the proceeds in a secure investment at, for example the LIBOR rate. Thus, the borrower is able to earn the difference between the LIBOR and lease rates, which is equivalent to GOFO.

The second strategy, a swap, allows the central bank to exchange gold for dollars. The swap is therefore a loan of dollars, secured with gold as collateral. Under the swap agreement, the central bank can invest the dollars at LIBOR while agreeing to pay the GOFO rate to the holder of the bullion. At the end of the leasing transaction, the gold is repurchased by the central bank. The central bank has earned the lease rate on the gold exchange and the bullion holder has earned the GOFO rate on his loaned dollars. As we should expect, the respective yields from the swap are equivalent to the yields earned in the normal leasing transaction.

Equation (6) can therefore be rewritten as:

$$IAB(t) = GOFO(t, T) - LIBOR(t, T) = w(t, T) - c(t, T) \quad (8)$$

where we have replaced  $R(t, T)$  with  $LIBOR(t, T)$ . Equations (3) and (8) lead to

$$lease = -IAB(t) = c(t, T) - w(t, T) \quad (9)$$

It can be seen from equation (9) that the lease rate may serve as a proxy for the convenience yield, particularly when  $w(t, T)$  is very small, which is the case in practice<sup>10</sup>.

Given the preceding analysis and under the assumption that storage costs are equal to zero, the price of a gold forward contract in terms of market variables can be expressed as:

$$\begin{aligned} f_0 e^{(lease)T} &= S_0 e^{(LIBOR)T} \\ f_0 &= S_0 e^{(GOFO)T} \end{aligned} \quad (10)$$

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<sup>10</sup> The cost of storing gold in a vault is indeed close to zero; hence the cost of storage is essentially equal to the cost of insurance (see Geman (2005)).

To compare the futures price calculated using equation (10) and the observed market futures prices, we introduce the difference  $M$  between the observed futures price and the theoretical forward price, both expressed by their logarithms

$$M = \log(futures) - \log(forward) \quad (11)$$

Note that we express the difference in terms of natural logarithms in order to reduce the heteroskedasticity in the  $M$  series. Table (III) shows summary statistics for the 1, 3, 6 and 12 month forward/futures series.

[Table III about here]

The mean values of the differences increase with the maturity from a low mean value of -0.0009 for the one month period to a high mean value of 0.19 for the 12 month one. In addition, the standard deviation increases with maturity in the same manner.

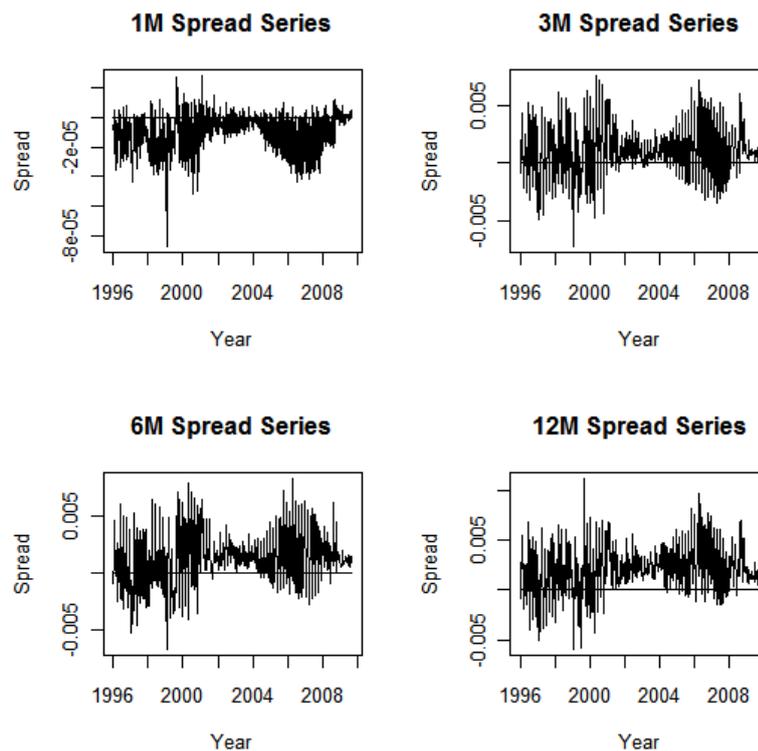


Figure 2: *PLOTS OF DIFFERENCES FUTURES/ FORWARDS FOR VARIOUS YEARS AND MATURITIES*  
 The figure shows the spreads between gold futures and forward for maturities of 1, 3 and 12 months. As the contract maturity increases, the spread gets wider, particularly in recent years. This may be indicative of risk-averse speculators demanding higher risk premiums for contracts of longer maturity.

Figure (2) depicts a plot of the four time series. We see that for the 1 month contract, the difference is mostly negative, indicating that calculated forward prices exceed observed

futures prices. However, with increasing maturities, observed futures prices exceed forward prices as calculated using equation (10). For the 12 month contract for instance, we see on the figure that the difference is mostly positive. This is in agreement with the previous findings of French (1983) and Park and Chen (1985).

It is known that increased speculation can affect futures prices. We therefore expect a relationship between the gold market speculative pressure and the futures/ forwards differences. Figure (3) exhibits the percentage differences versus the level of speculative pressure for the entire sample period.

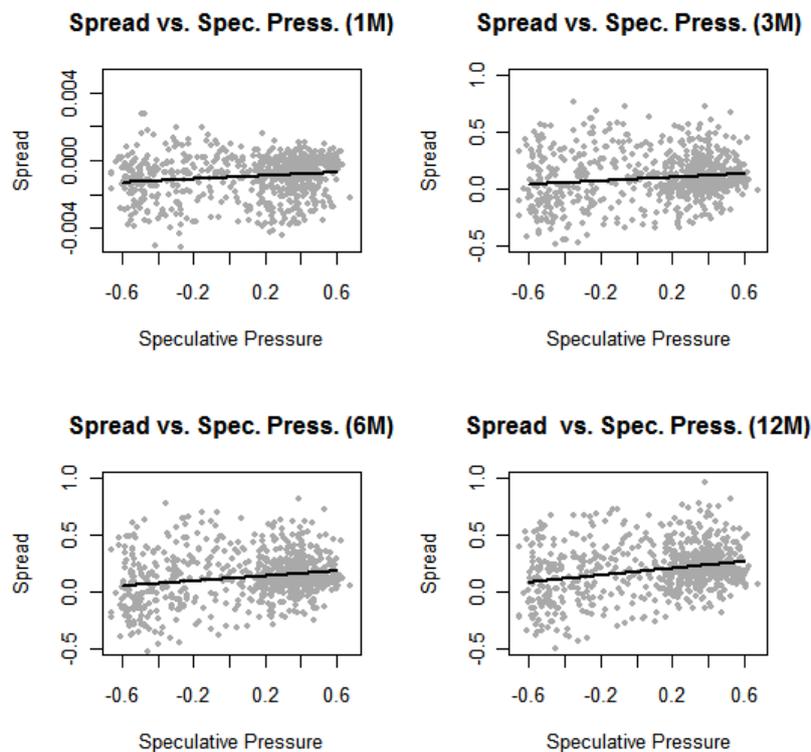


Figure 3: *PLOTS OF SPREADS FUTURES/ FORWARDS VS. THE LEVEL OF SPECULATIVE PRESSURE FOR VARIOUS MATURITIES*

The black line is a least squares regression fit to the sample data. The slope becomes increasingly positive with increasing time to maturity of the contracts, indicating that speculators require higher risk- premiums to act as counterparties in long term Futures.

In the plots, we have fitted a regression line to the data. As the rate tenor increases, the slope of the fitted regression lines becomes more positive. Equivalently, this implies that futures prices increasingly exceed forward prices as speculative pressure increases. If speculators operate in the long-term market, this suggests that there may be an increased risk premium attached to longer maturity futures contracts over the respective forward contract. This may be indicative of a preference by speculators for a higher risk-premium when speculating on the long-term price of gold, reflecting an increased risk-aversion when betting on prices in the distant future.

We therefore expect to find a relationship between the lease rate and the level of inventory. In order for the lease rate to remain small,  $w(t, T)$  must be close to zero.

A high lease rate or, equivalently, a high convenience yield under low storage costs, implies a high demand for physical gold and thus should have a negative coefficient in a regression of inventory level on the lease rate. Additionally, we should expect that the influence of the lease rate on inventory levels decreases with the increasing maturity of the lease. This is because short-term leasing will have a more immediate effect on inventory levels than long-term leases. Figure (4) shows the relationship between the lease rate and the level of discretionary (unallocated) inventory.

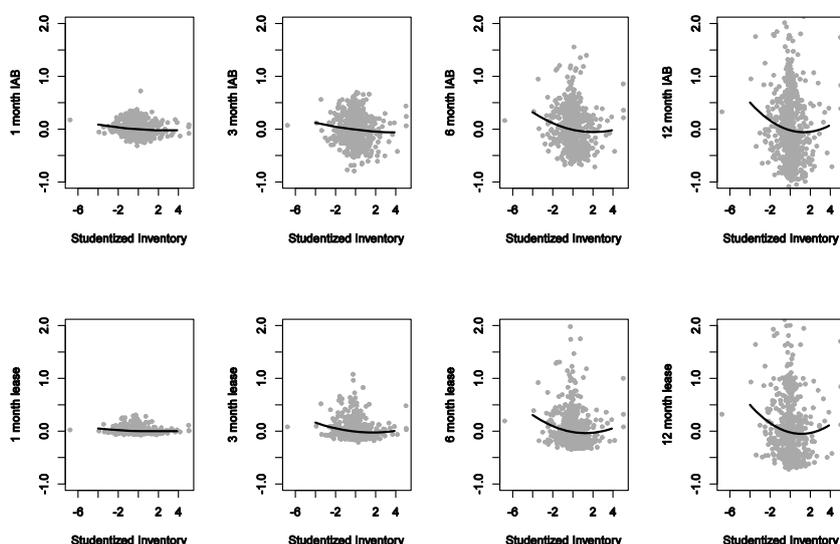


Figure 4: *RELATIONSHIP BETWEEN THE LEASE RATE, INTEREST-ADJUSTED SPREAD AND DISCRETIONARY INVENTORY LEVEL FOR THE YEARS 1996 – 2009*

The scatter plots illustrate the relationship between the lease rate and the level of discretionary inventory. For all lease durations we notice that the lease rate declines with increasing level of inventory. This is consistent with the idea that the lease rate can be considered as an observable form of the convenience yield of gold. Furthermore, data point dispersion increases with increasing lease rate tenor, suggesting that short-term leasing will have a more pronounced effect on inventory levels. The black line is a second-order polynomial fit to the data and is consistent with the shape of the convenience yield curve as discussed in Fama and French (1988). We note the similarity between the observable and inferred forms of the convenience yield as represented by the lease rate and interest-adjusted spread, respectively.

In a similar manner to the GOFO rate, we observe an asymmetric response of the convenience yield to positive and negative levels of discretionary inventory, but with one important difference. When the level of discretionary inventory is positive, the derived lease rate tends to zero as would be expected of the convenience yield. Conversely, for the GOFO rate we observe a concave upwards fitted curve, suggesting that for positive inventory levels, the GOFO rate does not decline to zero, but rather rises with increasing levels of inventory after exhibiting a local minimum when the level of discretionary inventory is close to zero.

To examine the relationship between the lease rate and the discretionary inventory, we use the linear regression model specified in equation (12):

$$Inv_t = \beta_1 Lease(i)_{t-1} + \beta_2 Inv_{t-1} + \varepsilon_t \quad (12)$$

Where  $Lease(i)_{t-1}$  is the prevailing  $i^{th}$  month lease rate in the previous time period, and  $Inv_{t-1}$  is the level of discretionary inventory at time  $t-1$ . The lagged inventory term is present in order to capture any residual autocorrelation. The results of the regression are given in table (IV).

[Table IV about here]

We can see that across all lease maturities, the coefficient  $\beta_1$  is negative. However, it is only highly significant for lease rates of one, three and six month maturities. For the twelve month rate, the coefficient is only weakly significant at the 10% level and is less than half the value of the 6 month coefficient. The magnitude and statistical significance of the coefficient diminishes with increasing maturity, being -0.11 for the 1 month rate and -0.05 for the 12 month lease duration. This reflects the diminishing impact of leasing activity on inventory levels as the lease tenor increases. The influence of discretionary inventory from the previous period is consistent across all maturities and the value of the coefficient remains close to 0.1, but is statistically insignificant. The negative lease rate coefficient and decreasing influence with maturity suggests that short term leasing activities act to reduce gold inventory levels. This occurs most likely because at time  $t$ , the bullion leased in previous time periods must be returned either to the central bank or the institution that leased the gold, the additional interest being charged in the form of bullion as opposed to dollars. Consequently, high lease rates in period  $t-1$  lead to repayments at time  $t$ , which, in turn, lead to a reduction in market inventories at time  $t$ . One plausible explanation for the declining influence with lease maturity is that, for the borrower, short-term lease rates tend to be lower than long-term rates, hence less expensive and resulting in increased short-term leasing activity.

Besides bullion leasing, there is another possible factor that can affect gold inventory levels. An increase in the number of open futures contracts may cause exchange inventory levels to increase in order to meet the possible delivery of physical gold at maturity. Therefore, as a robustness check, we test the hypothesis that speculative pressure results in increased inventory levels and, to that effect, specify a linear regression of the form shown in equation (13):

$$Inv_t = \alpha + \beta_1 Lease(i)_{t-1} + \beta_2 Inv_{t-1} + \beta_3 \psi_{t-1} + \varepsilon_t \quad (13)$$

Here  $Inv_t$  is the inventory level in millions of troy ounces at time  $t$ ,  $Lease(i)_{t-1}$  is the  $i^{th}$  month tenor derived lease rate at time  $t-1$ , and  $\psi_{t-1}$  is the speculative pressure at time  $t-1$  which we include in order to capture any dependence of the inventory on increased long

trader positions. Once again, lagged inventory is included in the regression in order to capture the significant lag-1 component of the inventory partial autocorrelation function. The results of regression (13) are shown in table (V).

[Table V about here]

The results in table (V) show that, by including the speculative pressure as an additional variable, the effect of the lease rate on discretionary inventory level decreases substantially. The results for the 1 month contract show that the influence of the lease rate has diminished in significance, the regression coefficient is now significant at the 10% level and has decreased in magnitude from a value of -0.11 to -0.07. In the case of the 1 month contract, the relation between the lease rate and the inventory level is now considerably weaker, significance being just outside the 5% level of confidence. For the 3, 6 and 12 month contracts, we conclude that there is no statistically significant relationship between the lease rate and the level of inventory.

Interestingly, although the lease rate relationship has weakened, there is now a positive and statistically significant relationship between the lagged speculative pressure and the discretionary inventory level. Furthermore, this relationship gradually increases with lease duration. The speculative pressure coefficient for the 1 month contract is 0.414 while that for the 12 month contract is 0.449. The lease rate and the speculative pressure appear to work in opposition to each other; the former acts to decrease short-term bullion inventories via lease repayments, while the latter result suggests speculators dominate leasing activity in the long-term. Finally, we note the continued presence of the carry-over effect such that the value of inventory at time  $t-1$  is positively related to inventory at time  $t$ . The results for the speculative pressure coefficient suggest that, due to increased speculative activity in long futures contracts, COMEX inventories have increased in order to cover the open contract positions.

## VI. Speculative Pressure: A VAR Model

To examine possible links between the differences futures-forwards and speculative activity, we employ a vector autoregression (VAR) model<sup>11</sup> to establish the dynamic relationship between the speculative pressure and the various difference series. The generalized reduced form specification of a VAR( $p$ ) model is given by equation (14).

$$\mathbf{r}_t = \phi_0 + \Phi_1 \mathbf{r}_{t-1} + \dots + \Phi_p \mathbf{r}_{t-p} + \varepsilon_t \quad (14)$$

---

<sup>11</sup> Cointegration tests were carried out between speculative pressure and the four difference series to determine if a VECM type model for cointegrated series was necessary. Using the Engle and Granger test, for all series the hypothesis of cointegration was rejected for the 1, 3, 6 and 12 month difference series and the speculative pressure. The tests may be provided on demand.

where  $\phi_0$  is a  $k \times 1$  vector,  $\Phi$  is a  $k \times k$  matrix, and  $\varepsilon_t$  is a serially uncorrelated random process with zero mean and positive definite variance-covariance matrix  $\Phi$  and  $p > 0$ . It is common in the literature to assume that  $\varepsilon_t$  is distributed as a multivariate normal. To build our model, we determine the optimal lag length,  $p$  using minimization of the Bayesian information criteria (BIC). The model is evaluated for multiple lag lengths and the length that corresponds to the smallest value of the information criteria is chosen as the lag length  $p$  for the VAR( $p$ ) model.

Using weekly data, for the 1, 3, 6 and 12 month contracts along with the speculative pressure, we test consecutively downwards from a maximum of  $p = 6$  lags, calculating the associated information criteria, to find the optimal lag length. For comparison, and to eliminate dependence on a single criterion, we employ three separate measures, the Akaike (AIC), the Schwarz (BIC) and the Hannin-Quinn (HQ) information criteria. If the individual criteria select different lag lengths, we first choose any lag length that is selected by two information criteria. If all three criteria differ, we defer to the BIC selection. Using this procedure, the information criteria select a lag of 3 for the 1 month series, and a lag of 2 for the 3, 6 and 12 month series.

With a lag length of  $p = 2$ , the VAR(2) model can be written in a more explicit bivariate form of equation (14). In terms of the speculative pressure  $\psi_t$  and the difference  $M_t$ , equation (14) can be written as

$$\begin{bmatrix} \psi_t \\ M_t \end{bmatrix} = \begin{bmatrix} \phi_{10} \\ \phi_{20} \end{bmatrix} + \begin{bmatrix} \Phi_{11}^1 & \Phi_{12}^1 \\ \Phi_{21}^1 & \Phi_{22}^1 \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ M_{t-1} \end{bmatrix} + \begin{bmatrix} \Phi_{11}^2 & \Phi_{12}^2 \\ \Phi_{21}^2 & \Phi_{22}^2 \end{bmatrix} \begin{bmatrix} \psi_{t-2} \\ M_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \quad (15)$$

The OLS estimated VAR coefficients are shown in table (VI).

[Table VI about here]

Looking at the  $\Phi$  matrices shown in the table, we see that for all maturities, the speculative pressure dynamics are driven by both lagged values of speculative pressure and the futures-forwards differences. Conversely, the effect of lagged speculative pressure on the level of difference is statistically significant only for the 3, 6 and 12 month series, suggesting there is a linear dependence between the width of the spread and the speculative pressure solely for maturities exceeding 1 month duration. Even though this dependence is exhibited only for long-term maturities, it is consistent with our finding that the relationship between futures contract returns and the speculative pressure is significant across all maturities: 1, 3, 6 and 12 months. For all maturities, both the lagged spread and lagged speculative pressure are significant determinants of the speculative pressure at time  $t$ .

To check the models, we test the residuals for serial correlation using the multivariate Ljung-Box statistic,  $Q_k(m)$ , where  $k$  is the dimension of  $\mathbf{r}_t$  and  $m$  is the number of lags used for the

test. This statistic is distributed asymptotically as  $\chi^2$  with  $mk^2 - g$  degrees of freedom,  $g$  being the number of parameters estimated in the VAR model coefficient matrices. For the 1-month series, the multivariate Ljung-Box statistic is 171.9 using 4 lags, with a  $p$ -value of less than 0.0001, suggesting that there is residual serial dependence in the bivariate return series. For the 3 month series, we have a value of 19.85 for the Ljung-Box test with an associated  $p$ -value of 0.3. Somewhat different results are obtained for the 6 month contract with a test statistic of 13.56 and  $p$ -value of 0.13. Finally, for the 12 month model, we note that  $Q_2(4) = 12.96$  with a  $p$ -value of 0.23, indicating that the model is sufficient at the 5% level.

## VII. The Effect of Lease Rates on Inventory Withdrawals

It is worthwhile investigating whether changes in the lease rate, or convenience yield, influence the level of bullion inventory and whether or not this changes with lease duration. While the gold lease rate is available primarily to agents and institutions participating in the over-the-counter (OTC) market, it should remain a viable proxy for overall gold market liquidity demand. Consequently, when the lease rate is high, it signals a period of high demand for bullion by the market. Under this assumption, we might expect to see increased inventory withdrawals along with increases in the derived lease rate.

Following Dincerler et al. (2005), we define withdrawals as the first-differenced series of discretionary inventory and regress inventory withdrawals on the change in the lease rate as specified in equation (16).

$$\Delta Inv_t = \beta_1 \Delta Lease(i)_t + \beta_2 \Delta Inv_{t-1} + \varepsilon_t \quad (16)$$

Table (VII) shows that there is a weak positive influence of changes in the lease rate on inventory withdrawals.

[Table VII about here]

Dependence on previous inventory levels is consistently negative and highly significant and has the rather intuitive interpretation that inventories are mean-reverting. The statistical significance of the results shows that there is a regular turnover of gold inventories in the futures market, but suggests that the quantity of leased gold remains independent of the lease rate level.

## VIII. The Growth Rate of Gold Forward vs. Gold Futures Prices

To compare the rate of increase of the futures price of gold with the market quoted GOFO rate, the Kalman filter is employed to estimate a model of the joint dynamics of gold futures prices and their rate of increase. The Kalman filter technique is quite efficient in a setting like ours where the second quantity is not directly observable (see Geman and Nguyen (2005)).

Hence, we construct a two-state variable model involving the spot price and the rate of growth of the gold futures price. We denote the gold spot price and its log by  $S_t$  and  $X_t$ , respectively and specify the model below.

We assume that the dynamics of the log spot price are driven under the real probability measure  $P$  by the following stochastic differential equation

$$dX_t = \left( \mu_t - \frac{\sigma^2}{2} + \sigma\lambda \right) dt + \sigma dW_t \quad (18)$$

and the dynamics of the growth rate  $\mu_t$  by the mean-reverting process

$$d\mu_t = \kappa(\alpha - \mu_t + \eta\xi) dt + \eta dB_t \quad (19)$$

where  $W$  and  $B$  are  $P$ -Brownian motions, the parameters  $\lambda$  and  $\xi$  respectively represent the market prices of log price and growth rate risk.

We are working in a complete market framework<sup>12</sup>. Consequently, this ensures the uniqueness of a risk-neutral measure  $Q$ , under which the dynamics of the spot price and growth rate have the reduced form below:

$$dX_t = \left( \mu_t - \frac{\sigma^2}{2} \right) dt + \sigma d\tilde{W}_t \quad (20)$$

$$d\mu_t = \kappa(\alpha - \mu_t) dt + \eta d\tilde{B}_t \quad (21)$$

The correlation coefficient  $\rho$  is given by:

$$\rho dt = d\tilde{W}_t d\tilde{B}_t$$

Within this two-state variable framework, the price  $F(X_t, \mu_t, t)$ , of the gold futures contract satisfies the following partial differential equation:

$$\frac{\partial F}{\partial t} + \mu_t \frac{\partial F}{\partial X_t} + \kappa(\alpha - \mu_t) \frac{\partial F}{\partial \mu_t} + \frac{1}{2} \sigma^2 \frac{\partial^2 F}{\partial X_t^2} + \rho \sigma \eta \frac{\partial^2 F}{\partial X_t \partial \mu_t} + \frac{1}{2} \eta^2 \frac{\partial^2 F}{\partial \mu_t^2} = 0 \quad (22)$$

This is subject to the terminal condition that the spot price is equal to the futures price at maturity  $T$ :

---

<sup>12</sup> COMEX trades at least 5 liquid gold futures contracts and our model has two sources of randomness. The number of traded instruments exceeds the number of sources of risk, so the market is complete.

$$F(X_T, \mu_T, T) = S(T) = e^{X_T} \quad (23)$$

Following Duffie and Kan (1996) and Geman-Nguyen (2005), we know that the solution to equation (22) can be written as the following exponential-affine function:

$$F_t^T = E_Q(S_T | \mathfrak{F}_t) = e^{A(\tau) + B(\tau)\mu_t} \quad (24)$$

where

$$A(\tau) = \left( -\frac{\alpha}{\kappa} + \frac{2\eta\lambda}{\kappa} + \frac{\eta^2}{\kappa^3} + \frac{\sigma\eta\rho}{\kappa^2} \right) e^{-\kappa\tau} \quad (25)$$

$$+ \left( -\alpha + \frac{\eta\lambda}{\kappa} + \frac{\eta^2}{2\kappa^2} + \frac{\sigma\eta\rho}{\kappa} \right) \tau + \frac{1}{\kappa^3} \left( \alpha\kappa^2 - \kappa\eta\lambda - \frac{3}{4}\eta^2 - \sigma\eta\rho\kappa \right)$$

$$B(\tau) = \frac{1 - e^{-\kappa\tau}}{\kappa} \quad (26)$$

In our model, these equations, in combination with equation (24), govern the price of a gold futures contract and depend upon six parameters:  $\alpha$ ,  $\kappa$ ,  $\lambda$ ,  $\sigma$ ,  $\eta$  and  $\rho$ . These parameters must be estimated from the observation data. One common method used for state space model estimation is the Kalman filter. Due to its ease of implementation and ubiquitous presence in the literature, it provides a convenient method by which to estimate and compare our model to existing models in the literature, for example that of Schwartz (1997).

To perform the filtering we use weekly observations of COMEX gold futures contracts for the first, third, sixth and twelfth nearby gold futures contract prices over the period of October 1986 to October 2009 yielding a total of 712 observations. At each observation date,  $t$ , we possess data for four contract maturities,  $\tau_i$ ,  $i = 1, \dots, 4$ . Our time series has been constructed such that the contract maturities at date  $t$  are constant and equal to  $\frac{1}{12}$ ,  $\frac{3}{12}$ ,  $\frac{6}{12}$  and  $\frac{12}{12}$ . Initialisation of the Kalman filter requires that we choose an initial parameter vector  $\varphi = \{\alpha, \kappa, \lambda, \sigma, \eta, \rho, |h_1|, |h_3|, |h_6|, |h_{12}|\}$ . Furthermore, we must initialise the variance-covariance matrix,  $H$  which is a diagonal matrix with elements  $h_1^2$ ,  $h_3^2$ ,  $h_6^2$ ,  $h_{12}^2$  on the diagonal. With these chosen parameters, we can estimate the parameters using maximum likelihood.

## IX. Kalman Filter and Estimation Results

After initialising the parameter vector and the Kalman filter matrices we ran the filter and obtained the optimised parameter set given in table (VIII).

[Table (VIII) about here ]

Table (VIII) displays the results of the log-likelihood estimation of the model parameters. The parameters  $\xi$ ,  $|h_2|$  and  $|h_3|$  are not statistically significant. However, all remaining parameters are significant at the <1% level. We observe that the market price of risk,  $\lambda$  is not statistically significant which is in agreement with the results of the Schwartz (1997) two-factor model which finds a statistically insignificant market price of risk for the periods of 1/2/85 to 6/13/95, 11/21/90 to 6/13/95 and 11/21/90 to 6/13/95.

Figures (5) and (6) graphically depict the results of the Kalman filter estimation of the joint system dynamics. In figure (5), we can observe a positive and consistent difference between the 3 month GOFO rate and the Kalman filtered asset growth rate. The figure suggests that since approximately 2003 until the end of 2007, the futures contract price has been priced consistently higher than the forward contract price. In terms of our model, this has the interpretation of an additional risk-premium in the value of the futures contract compared to the forward contract. It is interesting to note that the disappearance of the risk-premium coincides approximately with the beginning of the recent financial crisis.

This effect is depicted more clearly by the absolute difference between the filtered and quoted GOFO rate as illustrated by figure (6). The figure shows the absolute difference between the two series and the consistent positive difference can easily be seen beginning in the early part of 2003 and lasting until the end of 2007. It is this difference between the two growth rates that seems to be responsible for the degree of difference between the gold forward and futures contracts. This positive discrepancy between the two rates will be the subject of future research.

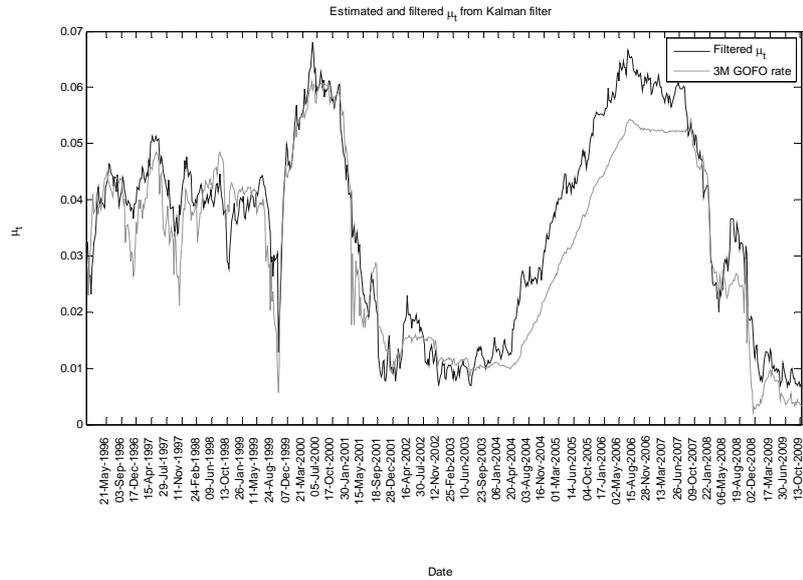


Figure 5: COMPARISON BETWEEN MARKET-QUOTED 3M GOFO RATE AND KALMAN ESTIMATED GROWTH RATE

The figure shows a comparison between the market quoted 3 month GOFO rate and the asset growth rate estimated using the Kalman filter. Starting in approximately February 2003 and ending in the latter half of 2007, there is an observable distinction between the GOFO rate and the filtered value suggesting a disappearance of the risk-premium that seems to coincide with the onset of the recent financial crisis.

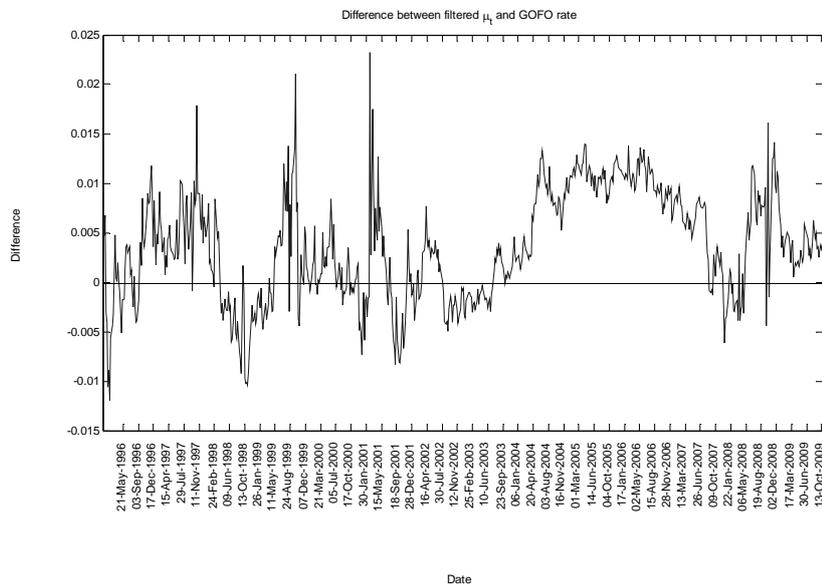


Figure 6: *DIFFERENCE BETWEEN THE 3 MONTH GOFO RATE AND THE KALMAN ESTIMATED ASSET GROWTH RATE.*

A positive difference between the two series can be observed beginning around February 2003 and persists over the remainder of the sample period.

## X. Conclusion

By examining the gold bullion leasing market over the period 1996-2009, we have argued that the derived lease rate can serve as an observable form of the convenience yield of gold. In such a framework, we exhibit a short-term relationship between the lease rate and the level of discretionary market inventory. In particular, we have shown that bullion leases of 1-month duration have a strong impact on inventory levels. Due to the nature of the gold leasing transactions, whereby interest on a bullion lease is repaid in the form of bullion, our results suggest that the 1-month lease rate has a negative effect on inventories such that lease repayments cause inventory levels to fall. However, this effect seems to be mitigated by the actions of speculators whose demand for long futures contracts results in increased inventory levels. Additionally, this speculative effect is seen to be consistently independent of the lease rate duration. Despite this, the lease rate does not appear to influence the size of inventory withdrawals. This could explain the mild influence of inventories on the price of gold (in contrast to most other storable commodities).

We have also shown that speculative pressure is positively and significantly related to futures returns. Controlling for price pressure, we find that while the price pressure hypothesis dominates speculative effects for futures returns calculated from short-term contracts, we cannot reject the hypothesis that long-term speculation has a positive effect on gold futures returns. That is, increased speculative activity in gold futures contracts is associated with higher futures returns.

Finally, our model has revealed an interesting decoupling of the GOFO rate and the growth rate of gold futures contract prices. In the period beginning late 2003 until the approximate onset of the recent crisis, there is evidence of a distinct and positive difference between the market quoted GOFO rate and the estimated growth rate, probably because of the arrival of new investors in the gold market. Historically, monetary authorities and hedging activities of gold producers used to dominate the gold market. The decreases in producer hedging activity and increased demand for gold as a financial asset have led to increased speculative activity in the bullion market. The subsequent recent participation and activities of gold ETFs may lead to an even more fundamental change in the gold futures market.

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<b>Coefficient</b>	<b>Estimate</b>	<b>Std. Err.</b>	<b>t-ratio</b>	<b>p-value</b>
<b>1M Returns</b>				
$\alpha$	0.0004	0.0008	0.51	0.61
$\beta_1$	0.0041	0.0502	0.08	0.93
$\beta_2$	0.0328	0.0381	0.86	0.39
$\beta_3$	0.0084	0.0021	3.90	<0.0001***
<b>3M Returns</b>				
$\alpha$	0.0004	0.0008	0.49	0.62
$\beta_1$	0.0108	0.0494	0.22	0.83
$\beta_2$	0.0366	0.0377	0.97	0.33
$\beta_3$	0.0084	0.0021	3.93	<0.0001***
<b>6M Returns</b>				
$\alpha$	0.0004	0.0008	0.48	0.63
$\beta_1$	0.0106	0.0498	0.21	0.83
$\beta_2$	0.0387	0.0376	1.03	0.30
$\beta_3$	0.0085	0.0021	3.94	<0.0001***
<b>12M Returns</b>				
$\alpha$	0.0004	0.0008	0.45	0.65
$\beta_1$	0.0083	0.0504	0.16	0.87
$\beta_2$	0.0459	0.0375	1.22	0.22
$\beta_3$	0.0086	0.0021	4.01	<0.0001***

Table I: *REGRESSION OF RETURNS ON SPECULATIVE PRESSURE FOR THE YEARS 1996 – 2009*

The table contains the estimation statistics of the regression model:

$r_{i,t} = \alpha + \beta_1 r_{i,t-1} + \beta_2 r_t^{S\&P500} + \beta_3 \psi_t + \varepsilon_t$ . The results demonstrate that current returns are not related to past returns. Additionally, we note that gold futures returns are positively, but insignificantly related to returns on the S&P 500 index. Additionally, we note that the level of speculative pressure is positively and significantly related to futures returns, suggesting that increased speculation leads to slightly higher returns, although the coefficient,  $\beta_3$  is rather small in magnitude, being on the order of 0.008

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1

<b>Coefficient</b>	<b>Estimate</b>	<b>Std. Err.</b>	<b>t-ratio</b>	<b>p-value</b>
<b>1M Returns</b>				
$\alpha$	0.0008	0.0008	1.00	0.32
$\beta_1$	-0.0483	0.0518	-0.93	0.35
$\beta_2$	0.0323	0.0338	0.95	0.35
$\beta_3$	0.0022	0.0008	2.76	0.006**
$\beta_4$	0.0097	0.0009	11.17	<0.0001***
<b>3M Returns</b>				
$\alpha$	0.0008	0.0008	0.97	0.33
$\beta_1$	-0.0434	0.0511	-0.85	0.40
$\beta_2$	0.0360	0.0338	1.07	0.29
$\beta_3$	0.0022	0.0008	2.78	0.006**
$\beta_4$	0.0098	0.0009	11.16	<0.0001***
<b>6M Returns</b>				
$\alpha$	0.0007	0.0008	0.95	0.34
$\beta_1$	-0.0436	0.0516	-0.84	0.40
$\beta_2$	0.0381	0.0336	1.13	0.26
$\beta_3$	0.0023	0.0008	2.81	0.005**
$\beta_4$	0.0097	0.0009	11.17	<0.0001***
<b>12M Returns</b>				
$\alpha$	0.0007	0.0008	0.92	0.36
$\beta_1$	-0.0451	0.0521	-0.86	0.39
$\beta_2$	0.0453	0.0333	1.36	0.18
$\beta_3$	0.0023	0.0008	2.89	0.004**
$\beta_4$	0.0096	0.0009	11.15	<0.0001***

Table II: *ROBUSTNESS CHECK FOR THE REGRESSION OF RETURNS ON SPECULATIVE PRESSURE AND PRICE PRESSURE FOR THE YEARS 1996 – 2009*

The table shows the results of a robust specification of the returns regression model given by equation (2) In this regression, we include both a speculative pressure term and a price pressure term, specified as the change in speculative pressure from time  $t - 1$  to time  $t$ . We divide both series by their respective standard deviation in order to compare them effectively. Results show that, after controlling for price pressure, the effect of speculative pressure on returns is mitigated. While still significant at the 1% level, the value of the speculative

pressure coefficient,  $\beta_3$ , is considerably reduced. However, we note that the speculative pressure effect does remain statistically significant at the <5% level across all returns series and increases with the maturity horizon. This is consistent with our previous findings that speculators may be induced to speculate long term as their increased risk-aversion for long-term bets resulting in higher futures contract risk-premiums.

<b>Variable</b>	<b>Statistic</b>
<b>1 Month Tenor</b>	
Mean	-0.0009
Std. Dev.	0.0012
Min	-0.0087
Median	-0.0007
Max	0.0028
<b>3 Month Tenor</b>	
Mean	0.0941
Std. Dev.	0.2129
Min	-0.7248
Median	0.0819
Max	0.7614
<b>6 Month Tenor</b>	
Mean	0.1294
Std. Dev.	0.2148
Min	-0.6706
Median	0.1228
Max	0.8221
<b>12 Month Tenor</b>	
Mean	0.1932
Std. Dev.	0.2268
Min	-0.5912
Median	0.1917
Max	1.1103

Table III: SUMMARY STATISTICS FOR THE 1, 3, 6 AND 12-MONTH SPREAD FOR THE YEARS 1996 - 2009

The table shows summary statistics for the series of the log-differences of observed futures prices and theoretical forward contract prices. Separate statistics are given for 1, 3, 6 and 12 month contract maturities. Although the 1 month contract exhibits a small degree of under-pricing of futures relative to forwards, we observe an increasingly positive mean in the 3, 6 and 12 month series, suggesting that, for these contracts, futures are over-priced, on average, relative to forwards.

<b>Coefficient</b>	<b>Estimate</b>	<b>Std. Err.</b>	<b>t-ratio</b>	<b>p-value</b>
<b>1M Lease Rate</b>				
$\beta_1$	-0.1068	0.0379	-2.82	0.005**
$\beta_2$	0.0916	0.0736	1.24	0.214
<b>3M Lease Rate</b>				
$\beta_1$	-0.0887	0.0353	-2.51	0.012*
$\beta_2$	0.0942	0.0745	1.26	0.207
<b>6M Lease Rate</b>				
$\beta_1$	-0.0682	0.0322	-2.12	0.034**
$\beta_2$	0.0977	0.0750	1.30	0.193
<b>12M Lease Rate</b>				
$\beta_1$	-0.0511	0.0284	-1.80	0.072.
$\beta_2$	0.1001	0.0747	1.34	0.181

Table IV REGRESSION OF INVENTORY LEVEL ON LAGGED LEASE RATE AND INVENTORY OVER THE PERIOD 1996-2009

Shown are the results of the regression  $Inv_t = \beta_1 Lease_{t-1} + \beta_2 Inv_{t-1} + \varepsilon_t$ . The lag-1 level of inventory was included to capture a possible autocorrelation. Across all maturities, the coefficient of lagged inventory remains positive, but statistically insignificant suggesting there is no carry-over inventory effect. Notably, the effect of the lagged lease rate diminishes with increasing lease rate duration. This suggests that short-term leasing has a more pronounced effect on current inventory levels. The negative coefficient implies that short-term lease repayments, in the form of physical bullion, act to reduce the current inventory level.

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1

<b>Coefficient</b>	<b>Estimate</b>	<b>Std. Err.</b>	<b>t-ratio</b>	<b>p-value</b>
<b>1M Lease Rate</b>				
$\beta_1$	-0.0650	0.0379	-1.71	0.08.
$\beta_2$	0.0680	0.0742	0.92	0.36
$\beta_3$	0.4139	0.1088	3.81	<0.001***
<b>3M Lease Rate</b>				
$\beta_1$	-0.0559	0.0354	-1.58	0.11
$\beta_2$	0.0683	0.0748	0.91	0.36
$\beta_3$	0.4283	0.1095	3.91	<0.0001***
<b>6M Lease Rate</b>				
$\beta_1$	-0.0426	0.0326	-1.58	0.19
$\beta_2$	0.0696	0.0750	0.93	0.35
$\beta_3$	0.4416	0.1089	4.05	<0.0001***
<b>12M Lease Rate</b>				
$\beta_1$	-0.0316	0.0285	-1.11	0.27
$\beta_2$	0.0705	0.0746	0.95	0.34
$\beta_3$	0.4498	0.1071	4.20	<0.0001***

Table V: REGRESSION OF INVENTORY AGAINST LEASE RATE AND SPECULATIVE PRESSURE OVER THE PERIOD 1996-2009

The table shows the coefficient estimates and associated regression statistics for the regression model:  $Inv_t = \alpha + \beta_1 Lease(i)_{t-1} + \beta_2 Inv_{t-1} + \beta_3 \psi_{t-1} + \varepsilon_t$ . By controlling for speculative pressure, we note that the effect of the lease rate on discretionary inventory level is now very weak and significant at just under the 10% level of significance for the 1 month rate tenor. This suggests that speculative pressure increases inventory levels counteracting the negative effect of lease repayments.

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1

**Panel A: VAR Model Estimation**

Tenor	Parameter	$\phi_0$	$\Phi_1$		$\Phi_2$	
1 Month	Coefficient	0.0056	1.1557	6.4058	-0.1896	-4.3062
		-0.0008	0.0005	0.2930	-0.0002	-0.0737
	Std. Err.	0.0047	0.0364	2.6700	0.0364	2.6784
		0.0001	0.0005	0.0376	0.0005	0.0377
3 Month	Coefficient	0.0050	1.1552	0.0298	-0.1868	-0.0474
		0.0556	0.1829	0.5022	-0.1355	-0.1474
	Std. Err.	0.0037	0.0363	0.0169	0.0363	0.0169
		0.0081	0.0799	0.0372	0.0800	0.0372
6 Month	Coefficient	0.0051	1.1559	0.0348	-0.1875	-0.0488
		0.0711	0.1982	0.5149	-0.1331	-0.1181
	Std. Err.	0.0039	0.0362	0.0171	0.0363	0.0171
		0.0085	0.0792	0.0373	0.0793	0.0373
12 Month	Coefficient	0.0043	1.1584	0.0369	-0.1909	-0.0411
		0.1028	0.2404	0.5008	-0.1489	-0.0840
	Std. Err.	0.0045	0.0363	0.0165	0.0364	0.0164
		0.0102	0.0825	0.0375	0.0828	0.0373

Table VI: VAR(2) MODEL ESTIMATION RESULTS

The table shows the OLS estimated coefficients along with their standard errors for the VAR(2) model defined in the equation:

$$\begin{bmatrix} \psi_t \\ M_t \end{bmatrix} = \begin{bmatrix} \phi_{10} \\ \phi_{20} \end{bmatrix} + \begin{bmatrix} \Phi_{11}^1 & \Phi_{12}^1 \\ \Phi_{21}^1 & \Phi_{22}^1 \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ M_{t-1} \end{bmatrix} + \begin{bmatrix} \Phi_{11}^2 & \Phi_{12}^2 \\ \Phi_{21}^2 & \Phi_{22}^2 \end{bmatrix} \begin{bmatrix} \psi_{t-2} \\ M_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

<b>Coefficient</b>	<b>Estimate</b>	<b>Std. Err.</b>	<b>t-ratio</b>	<b>p-value</b>
<b>1M Lease Rate</b>				
$\beta_1$	0.1068	0.1063	1.00	0.32
$\beta_2$	-0.4988	0.0504	-9.90	<0.0001***
<b>3M Lease Rate</b>				
$\beta_1$	0.1198	0.1424	0.84	0.40
$\beta_2$	-0.4982	0.0506	-9.85	<0.0001***
<b>6M Lease Rate</b>				
$\beta_1$	0.2265	0.1914	1.18	0.24
$\beta_2$	-0.4978	0.0507	-9.82	<0.0001***
<b>12M Lease Rate</b>				
$\beta_1$	0.1901	0.2421	0.79	0.43
$\beta_2$	-0.4977	0.0509	-9.79	<0.0001***

Table VII: *REGRESSION OF WITHDRAWALS ON CHANGES IN THE LEASE RATE OVER THE PERIOD 1996-2009*

The table shows the regression output for the model:  $\Delta Inv_t = \beta_1 \Delta Lease(i)_t + \beta_2 \Delta Inv_{t-1} + \varepsilon_t$ . This examines the effect of the lease rate on inventory withdrawals, while controlling for past inventory withdrawals. The coefficient  $\beta_2$  remains nearly constant in value, negative and statistically significant across all lease rates. Significance codes: 0 '\*\*\*', 0.001 '\*\*', 0.01 '\*', 0.05 '.', 0.1

<b>Parameter</b>	<b>Estimation</b>	<b>Std. Err.</b>	<b><i>t</i>-value</b>	<b><i>p</i>-value</b>
$\sigma$	1.3618	0.0313	43.5138	0.0000
$\kappa$	0.2396	0.0245	9.7779	0.0000
$\alpha$	0.0248	0.0167	1.4805	0.1392
$\lambda$	0.0000	0.2853	0.0000	1.0000
$\eta$	0.0163	0.0006	25.3381	0.0000
$\rho$	0.4255	0.0324	13.1239	0.0000
$h_1$	-0.0024	0.0001	36.1522	0.0000
$h_3$	0.0000	0.0001	-0.0001	0.9999
$h_6$	-0.0005	0.0000	-32.3597	0.0000
$h_{12}$	0.0003	0.0001	3.9473	0.0001
<b>Log-Likelihood</b>		-117788.7		

Table VIII: KALMAN FILTER ESTIMATION RESULTS  
Parameter vector optimization results from the log-likelihood maximization.

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